Option D Astrophysics

D1 Stellar quantities

This section begins with a brief description of the various objects that comprise the universe, especially stars. We discuss astronomical distances and the main characteristics of stars: their luminosity and apparent brightness. Table **D.1** presents a summary of key terms.

Learning objectives

- Describe the main objects comprising the universe.
- Describe the nature of stars.
- Understand astronomical distances.
- Work with the method of parallax.
- Define luminosity and apparent brightness and solve problems with these quantities and distance.

D1.1 Objects in the universe

We live in a part of space called the **solar system**: a collection of eight major planets (Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus and Neptune) bound in elliptical orbits around a star we call the Sun. Pluto has been stripped of its status as a major planet and is now called a 'dwarf planet'. The orbit of the Earth is almost circular; that of Mercury is the most elliptical. All planets revolve around the Sun in the same direction. This is also true of the comets, with a few exceptions, the most famous being Halley's comet. All the planets except Mercury and Venus have moons orbiting them.

Leaving the solar system behind, we enter interstellar space, the space between stars. At a distance of 4.2 light years (a light year is the distance travelled by light in one year) we find Proxima Centauri, the nearest star to us after the Sun. Many stars find themselves in **stellar clusters**, groupings of large numbers of stars that attract each other gravitationally and are relatively close to one another. Stellar clusters are divided into two groups: **globular clusters**, containing large numbers of mainly old, evolved stars, and **open clusters**, containing smaller numbers of young stars (some are very hot) that are further apart, Figure **D.1**.

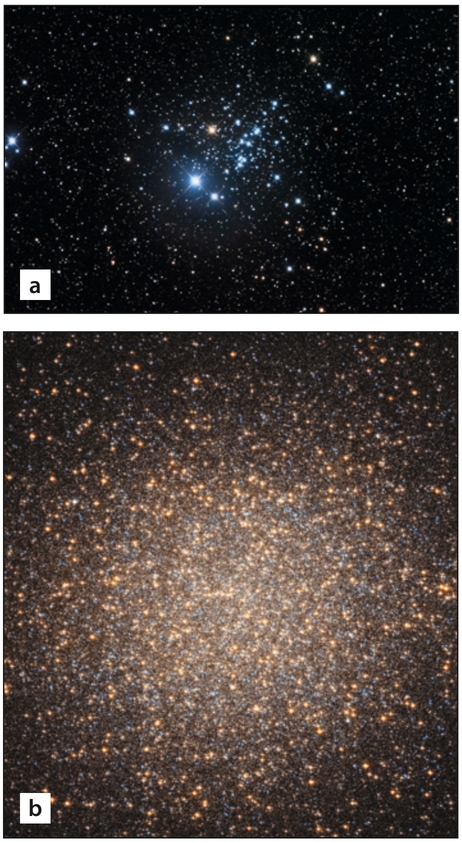


Figure D.1 **a** The open cluster M36; **b** the globular cluster M13.

Very large numbers of stars and stellar clusters (about 200 billion of them) make up our **galaxy**, the Milky Way, a huge assembly of stars that are kept together by gravity. A galaxy with spiral arms (similar to the one in Figure **D.2a**), it is about 120 000 light years across; the arm in which our solar system is located can be seen on a clear dark night as the spectacular 'milky' glow of millions of stars stretching in a band across the sky.

As we leave our galaxy behind and enter intergalactic space, we find that our galaxy is part of a group of galaxies – a **cluster** (such as the one shown in Figure **D.2b**), known as the Local Group. There are about 30 galaxies in the Local Group, the nearest being the Large Magellanic

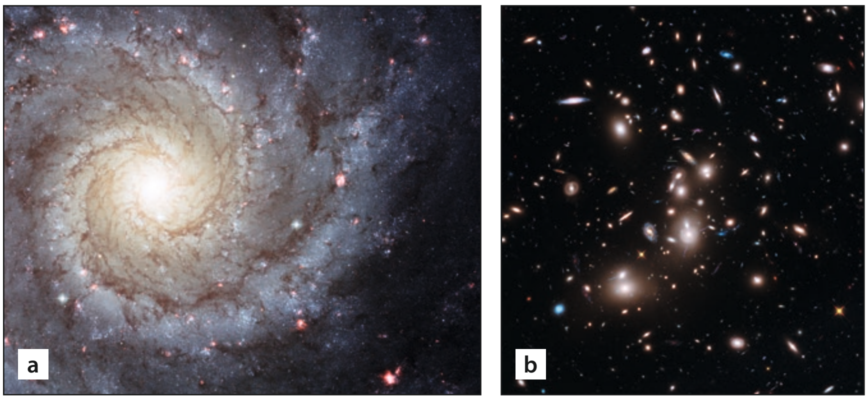


Figure D.2 **a** The spiral galaxy M74; **b** the galaxy cluster Abell 2744.

Cloud at a distance of about 160 000 light years. In this group, we also find the Andromeda galaxy, a spiral galaxy like our own and the largest member of the Local Group. Andromeda is expected to collide with the Milky Way in 4 billion years or so.

As we move even further out, we encounter collections of clusters of galaxies, known as **superclusters**. If we look at the universe on a really large scale, more than 10^8 light years, we then see an almost uniform distribution of matter. At such enormous scales, every part of the universe looks the same.

Binary star	Two stars orbiting a common centre
Black dwarf	The remnant of a white dwarf after it has cooled down. It has very low luminosity
Black hole	A singularity in space time; the end result of the evolution of a very massive star
Brown dwarf	Gas and dust that did not reach a high enough temperature to initiate fusion. These objects continue to compact and cool down
Cepheid variable	A star of variable luminosity. The luminosity increases sharply and falls off gently with a well-defined period. The period is related to the absolute luminosity of the star and so can be used to estimate the distance to the star
Cluster of galaxies	Galaxies close to one another and affecting one another gravitationally, behaving as one unit
Comet	A small body (mainly ice and dust) orbiting the Sun in an elliptical orbit
Constellation	A group of stars in a recognisable pattern that *appear* to be near each other in space
Dark matter	Generic name for matter in galaxies and clusters of galaxies that is too cold to radiate. Its existence is inferred from techniques other than direct visual observation
Galaxy	A collection of a very large number of stars mutually attracting one another through the gravitational force and staying together. The number of stars in a galaxy varies from a few million in dwarf galaxies to hundreds of billions in large galaxies. It is estimated that 100 billion galaxies exist in the observable universe
Interstellar medium	Gases (mainly hydrogen and helium) and dust grains (silicates, carbon and iron) filling the space between stars. The density of the interstellar medium is very low. There is about one atom of gas for every cubic centimetre of space. The density of dust is a trillion times smaller. The temperature of the gas is about 100 K
Main-sequence star	A normal star that is undergoing nuclear fusion of hydrogen into helium. Our Sun is a typical main-sequence star
Nebula	Clouds of 'dust', i.e. compounds of carbon, oxygen, silicon and metals, as well as molecular hydrogen, in the space in between stars
Neutron star	The end result of the explosion of a red supergiant; a very small star (a few tens of kilometres in diameter) and very dense. This is a star consisting almost entirely of neutrons. The neutrons form a superfluid around a core of immense pressure and density. A neutron star is an astonishing macroscopic example of microscopic quantum physics
Planetary nebula	The ejected envelope of a red giant star
Red dwarf	A very small star with low temperature, reddish in colour
Red giant	A main-sequence star evolves into a red giant – a very large, reddish star. There are nuclear reactions involving the fusion of helium into heavier elements
Stellar cluster	A group of stars that are physically near each other in space, created by the collapse of a single gas cloud
Supernova (Type Ia)	The explosion of a white dwarf that has accreted mass from a companion star exceeding its stability limit
Supernova (Type II)	The explosion of a red supergiant star: The amount of energy emitted in a supernova explosion can be staggering – comparable to the total energy radiated by our Sun in its entire lifetime!
White dwarf	The end result of the explosion of a red giant. A small, dense star (about the size of the Earth), in which no nuclear reactions take place. It is very hot but its small size gives it a very low luminosity

Table D.1 Definitions of terms.

Worked example

D.1 Take the density of interstellar space to be one atom of hydrogen per cm^3 of space. How much mass is there in a volume of interstellar space equal to the volume of the Earth? Give an order-of-magnitude estimate without using a calculator.

The volume of the Earth is

$$V \approx \tfrac{4}{3}\pi R^3$$
$$\approx \tfrac{4}{3} \times 3 \times (6 \times 10^6)^3\,\text{m}^3$$
$$\approx 4 \times 200 \times 10^{18}$$
$$\approx 10^{21}\,\text{m}^3$$

The number of atoms in this volume is $10^{21} \times 10^6 = 10^{27}$ atoms of hydrogen (one atom in a cubic cm implies 10^6 atoms in a cubic metre). This corresponds to a mass of

$$10^{27} \times 10^{-27}\,\text{kg} \approx 1\,\text{kg}.$$

D1.2 The nature of stars

A star such as our own Sun radiates an enormous amount of power into space – about $10^{26}\,\text{J}\,\text{s}^{-1}$. The source of this energy is nuclear fusion in the interior of the star, in which nuclei of hydrogen fuse to produce helium and energy. Because of the **high temperatures** in the interior of the star, the electrostatic repulsion between protons can be overcome, allowing hydrogen nuclei to come close enough to each other to fuse. Because of the **high pressure** in stellar interiors, the nuclei are sufficiently close to each other to give a high probability of collision and hence fusion. The sequence of nuclear fusion reactions that take place is called the **proton–proton cycle** (Figure **D.3**):

$$^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^0_1\text{e}^+ + \nu$$

$$^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$$

$$^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2{}^1_1\text{H}$$

The net result of these reactions is that four hydrogen nuclei turn into one helium nucleus (to see this multiply the first two reactions by 2 and add side by side). Energy is released at each stage of the cycle, but most of it is released in the third and final stage. The energy produced is carried away by the photons (and neutrinos) produced in the reactions. As the photons move outwards they collide with the surrounding material, creating a **radiation pressure** that opposes the gravitational pressure arising from the mass of the star. In the outer layers, convection currents also carry the energy outwards. In this way, the balance between radiation and gravitational pressures keeps the star in equilibrium (Figure **D.4**).

Nuclear fusion provides the energy that is needed to keep the star hot, so that the radiation pressure is high enough to oppose further gravitational contraction, and at the same time to provide the energy that the star is radiating into space.

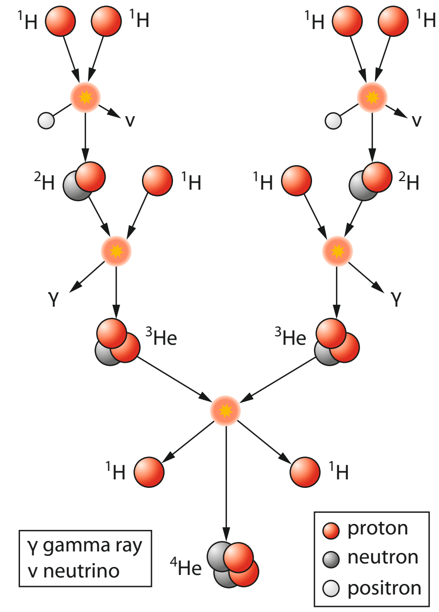


Figure D.3 The proton–proton cycle of fusion reactions.

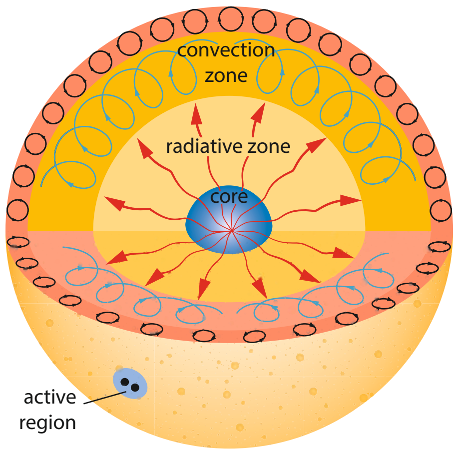


Figure D.4 The stability of a star depends on equilibrium between two opposing forces: gravitation, which tends to collapse the star, and radiation pressure, which tends to make it expand.

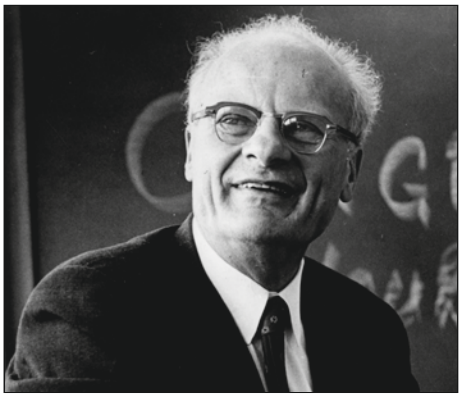


Figure D.5 Hans Bethe, who unravelled the secrets of energy production in stars.

Exam tip
$1\,\text{AU} = 1.5 \times 10^{11}\,\text{m}$
$1\,\text{ly} = 9.46 \times 10^{15}\,\text{m}$
$1\,\text{pc} = 3.09 \times 10^{16}\,\text{m} = 3.26\,\text{ly}$

Many of the details of nuclear fusion reactions in stars were worked out by the legendary Cornell physicist Hans Bethe (1906–2005) (Figure **D.5**).

D1.3 Astronomical distances

In astrophysics, it is useful to have a more convenient unit of distance than the metre!

A **light year** (ly) is the distance travelled by light in one year. Thus:

$$1\,\text{ly} = 3 \times 10^8 \times 365 \times 24 \times 60 \times 60\,\text{m}$$
$$= 9.46 \times 10^{15}\,\text{m}$$

Also convenient for measuring large distances is the **parsec** (pc), a unit that will be properly defined in Section **D1.4**:

$$1\,\text{pc} = 3.26\,\text{ly} = 3.09 \times 10^{16}\,\text{m}$$

Yet another convenient unit is the **astronomical unit** (AU), which is the average radius of the Earth's orbit around the Sun:

$$1\,\text{AU} = 1.5 \times 10^{11}\,\text{m}$$

The average distance between stars in a galaxy is about 1 pc. The distance to the nearest star (Proxima Centauri) is approximately 4.2 ly = 1.3 pc. A simple message sent to a civilisation on Proxima Centauri would thus take 4.2 yr to reach it and an answer would arrive on Earth another 4.2 yr later.

The average distance between galaxies varies from about 100 kiloparsecs (kpc) for galaxies within the same cluster to a few megaparsecs (Mpc) for galaxies belonging to different clusters.

Worked examples

D.2 The Local Group is a cluster of some 30 galaxies, including our own Milky Way and the Andromeda galaxy. It extends over a distance of about 1 Mpc. Estimate the average distance between the galaxies of the Local Group.

Assume that a volume of

$$V \approx \tfrac{4}{3}\pi R^3$$
$$\approx \tfrac{4}{3} \times 3 \times (0.5)^3\,\text{Mpc}^3$$
$$\approx 0.5\,\text{Mpc}^3$$

is uniformly shared by the 30 galaxies. Then to each there corresponds a volume of

$$\frac{0.5}{30}\text{Mpc}^3 = 0.017\,\text{Mpc}^3$$

The linear size of each volume is thus

$$\sqrt[3]{0.017\,\text{Mpc}^3} \approx 0.3\,\text{Mpc}$$
$$= 300\,\text{kpc}$$

so we may take the average separation of the galaxies to be about 300 kpc.

D.3 The Milky Way galaxy has about 2×10^{11} stars. Assuming an average stellar mass equal to that of the Sun, estimate the mass of the Milky Way.

The mass of the Sun is 2×10^{30} kg, so the Milky Way galaxy has a mass of about

$$2 \times 10^{11} \times 2 \times 10^{30}\,\text{kg} = 4 \times 10^{41}\,\text{kg}$$

D.4 The observable universe contains some 100 billion galaxies. Assuming an average galactic mass comparable to that of the Milky Way, estimate the mass of the observable universe.

The mass is $100 \times 10^{9} \times 4 \times 10^{41}\,\text{kg} = 4 \times 10^{52}\,\text{kg}$.

D1.4 Stellar parallax and its limitations

The **parallax** method is a means of measuring astronomical distances. It takes advantage of the fact that, when an object is viewed from two different positions, it appears displaced relative to a fixed background. If we measure the angular position of a star and then repeat the measurement some time later, the two positions will be different relative to a background of very distant stars, because in the intervening time the Earth has moved in its orbit around the Sun. We make two measurements of the angular position of the star six months apart; see Figure **D.6**.

The distance between the two positions of the Earth is $D = 2R$, the diameter of the Earth's orbit around the Sun ($R = 1.5 \times 10^{11}$ m). The distance d to the star is given by

$$\tan p = \frac{R}{d} \quad \Rightarrow \quad d = \frac{R}{\tan p}$$

Since the parallax angle is very small, $\tan p \approx p$, where the parallax p is measured in radians, and so $d = \dfrac{R}{p}$.

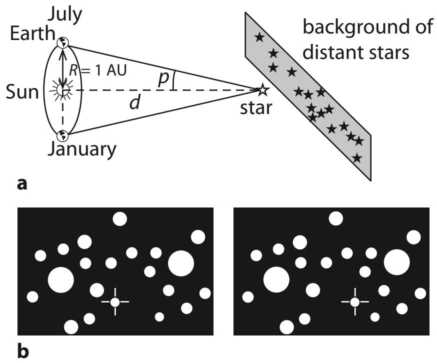


Figure D.6 **a** The parallax of a star. **b** Two 'photographs' of the same region of the sky taken six months apart. The position of the star (indicated by a cross) has shifted, relative to the background stars, in the intervening six months.

> The parallax angle (shown in Figure **D.6**) is the angle, at the position of the star, that is subtended by a distance equal to the radius of the Earth's orbit around the Sun (1 AU).

The parallax method can be used to define a common unit of distance in astronomy, the **parsec**. One parsec (from **par**allax **sec**ond) is the distance to a star whose parallax is 1 arc second, as shown in Figure **D.7**. An arc second is 1/3600 of a degree.

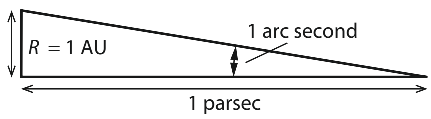


Figure D.7 Definition of a parsec: 1 parsec is the distance at which 1 AU subtends an angle of 1 arc second.

In conventional units,

$$1\,\text{pc} = \frac{1\,\text{AU}}{1''} = \frac{1.5 \times 10^{11}}{\left(\frac{2\pi}{360}\right)\left(\frac{1}{3600}\right)}\,\text{m} = 3.09 \times 10^{16}\,\text{m}$$

(The factor of $\dfrac{2\pi}{360}$ converts degrees to radians.)

> If the parallax of a star is known to be p arc seconds, its distance is
>
> $$d\ (\text{in parsecs}) = \frac{1}{p}\ (\text{in arc seconds}).$$

Exam tip

You will not be asked to provide these derivations in an exam. You should just know that d (in parsecs) $= \dfrac{1}{p}$ (in arc seconds).

You must also understand the limitations of this method.

Ancient methods are still useful!

Astrophysicists still use an ancient method of measuring the apparent brightness of stars. This is the **magnitude scale**. Given a star of apparent brightness b, we assign to that star an **apparent magnitude** m, defined by

$$\frac{b}{b_0} = 100^{-m/5}$$

The value $b_0 = 2.52 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ is taken as the reference value for apparent brightness. Taking logarithms (to base 10) gives the equivalent form

$$m = -\frac{5}{2}\log\left(\frac{b}{b_0}\right)$$

Since $100^{1/5} = 2.512$, the first equation above can also be written as

$$\frac{b}{b_0} = 2.512^{-m}$$

If the star is too far away, however, the parallax angle is too small to be measured and this method fails. Typically, measurements from observatories on Earth allow distances up to 100 pc to be determined by the parallax method, which is therefore mainly used for nearby stars. Using measurements from satellites without the distortions caused by the Earth's atmosphere (turbulence, and variations in temperature and refractive index), much larger distances can be determined using the parallax method. The Hipparcos satellite (launched by ESA, the European Space Agency, in 1989) measured distances to stars 1000 pc away; Gaia, launched by ESA in December 2013, is expected to do even better, extending the parallax method to distances beyond 100 000 pc!

Table **D.2** shows the five nearest stars (excluding the Sun).

Star	Distance/ly
Proxima Centauri	4.3
Barnard's Star	5.9
Wolf 359	7.7
Lalande 21185	8.2
Sirius	8.6

Table D.2 Distances to the five nearest stars.

D1.5 Luminosity and apparent brightness

Stars are assumed to radiate like black bodies. For a star of surface area A and absolute surface temperature T, we saw in Topic **8** that the power radiated is

$$L = \sigma A T^4$$

where the constant σ is the Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$).

The power radiated by a star is known in astrophysics as the **luminosity**. It is measured in watts.

Consider a star of luminosity L. Imagine a sphere of radius d centred at the location of the star. The star radiates uniformly in all directions, so the energy radiated in 1 s can be thought of as distributed over the surface of this imaginary sphere. A detector of area a placed somewhere on this sphere will receive a small fraction of this total energy (see Figure **D.8a**).

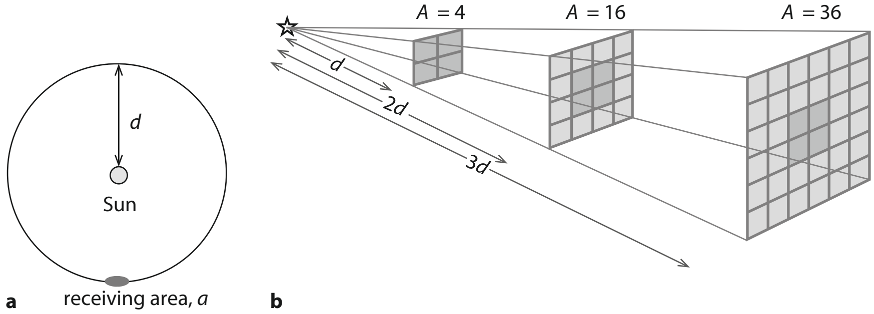


Figure D.8 **a** The Sun's energy is distributed over an imaginary sphere of radius equal to the distance between the Sun and the observer. The observer thus receives only a very small fraction of the total energy, equal to the ratio of the receiver's area to the total area of the imaginary sphere. **b** The inverse square law.

Exam tip
In many problems you will need to know that the surface area of a sphere of radius R is $A = 4\pi R^2$.

This fraction is equal to the ratio of the detector area a to the total surface area of the sphere; that is, the received power is $\frac{aL}{4\pi d^2}$.

This shows that the apparent brightness is directly proportional to the luminosity, and varies as the inverse square of the star's distance (see Figure **D.8b**). By combining the formula for luminosity with that for apparent brightness, we see that

$$b = \frac{\sigma A T^4}{4\pi d^2}$$

Apparent brightness is easily measured (with a charge-coupled device, or CCD). If we also know the distance to a star, then we can determine its luminosity. Knowing the luminosity of a star is important because it tells a lot about the nature of the star.

The received power per unit area is called the **apparent brightness** and is given by

$$b = \frac{L}{4\pi d^2}$$

The unit of apparent brightness is $\mathrm{W\,m^{-2}}$.

Exam tip
Apparent brightness in astrophysics is what is normally called intensity in physics.

Worked examples

D.5 The radius of star A is three times that of star B and its temperature is double that of B. Find the ratio of the luminosity of A to that of B.

$$\frac{L_A}{L_B} = \frac{\sigma 4\pi (R_A)^2 T_A{}^4}{\sigma 4\pi (R_B)^2 T_B{}^4}$$

$$= \frac{(R_A)^2 T_A{}^4}{(R_B)^2 T_B{}^4}$$

$$= \frac{(3R_B)^2 (2T_B)^4}{(R_B)^2 T_B{}^4}$$

$$= 3^2 \times 2^4 = 144$$

D.6 The stars in Worked example **D.5** have the same apparent brightness when viewed from the Earth. Calculate the ratio of their distances.

$$\frac{b_A}{b_B}=1$$

$$=\frac{L_A/(4\pi d_A{}^2)}{L_B/(4\pi d_B{}^2)}$$

$$=\frac{L_A}{L_B}\frac{d_B{}^2}{d_A{}^2}$$

$$\Rightarrow \quad \frac{d_A}{d_B}=12$$

D.7 The apparent brightness of a star is $6.4\times10^{-8}\,\mathrm{W\,m^{-2}}$. Its distance is 15 ly. Find its luminosity.

We use $b=\dfrac{L}{4\pi d^2}$ to find

$$L=b4\pi d^2=\left(6.4\times10^{-8}\frac{\mathrm{W}}{\mathrm{m}^2}\right)\times4\pi\times(15\times9.46\times10^{15})^2\,\mathrm{m}^2$$

$$=1.6\times10^{28}\,\mathrm{W}$$

D.8 A star has half the Sun's surface temperature and 400 times its luminosity. Estimate the ratio of the star's radius to that of the Sun. The symbol $R_\odot$ stands for the radius of the Sun.

We have that

$$400=\frac{L}{L_\odot}=\frac{\sigma4\pi(R^2)T^4}{\sigma4\pi(R_\odot)T_\odot{}^4}=\frac{(R^2)(T_\odot/2)^4}{(R_\odot)^2T_\odot{}^4}=\frac{R^2}{(R_\odot)^2 16}$$

$$\Rightarrow \quad \frac{R^2}{(R_\odot)^2}=16\times400$$

$$\Rightarrow \quad \frac{R}{R_\odot}=80$$

Nature of science

Reasoning about the universe

Over millennia, humans have mapped the planets and stars, recording their movements and relative brightness. By applying the simple idea of parallax, the change in position of a star in the sky at times six months apart, astronomers could begin to measure the distances to stars. Systematic measurement of the distances and the relative brightness of stars and galaxies, with increasingly sophisticated tools, has led to an understanding of a universe that is so large it is difficult to imagine.

Test yourself

1 Determine the distance to Procyon, which has a parallax of 0.285″.

2 The distance to Epsilon Eridani is 10.8 ly. Calculate its parallax angle.

3 Betelgeuse has an angular diameter of 0.016″ (that is, the angle subtended by the star's diameter at the eye of an observer) and a parallax of 0.0067″.
   a Determine the distance of Betelgeuse from the Earth.
   b What is its radius in terms of the Sun's radius?

4 A neutron star has an average density of about $10^{17}\,\mathrm{kg\,m^{-3}}$. Show that this is comparable to the density of an atomic nucleus.

5 A sunspot near the centre of the Sun is found to subtend an angle of 4.0 arc seconds. Find the diameter of the sunspot.

6 The resolution of the Hubble Space Telescope is about 0.05 arc seconds. Estimate the diameter of the smallest object on the Moon that can be resolved by the telescope. The Earth–moon distance is $D = 3.8 \times 10^{8}\,\mathrm{m}$.

7 The Sun is at a distance of 28 000 ly from the centre of the Milky Way and revolves around the galactic centre with a period of about 211 million years. Estimate from this information the orbital speed of the Sun and the total mass of the Milky Way.

8 **a** Describe, with the aid of a clear diagram, what is meant by the **parallax method** in astronomy.
   b Explain why the parallax method fails for stars that are very far away.

9 The light from a star a distance of 70 ly away is received on Earth with an apparent brightness of $3.0 \times 10^{-8}\,\mathrm{W\,m^{-2}}$. Calculate the luminosity of the star.

10 The luminosity of a star is $4.5 \times 10^{28}\,\mathrm{W}$ and its distance from the Earth is 88 ly. Calculate the apparent brightness of the star.

11 The apparent brightness of a star is $8.4 \times 10^{-10}\,\mathrm{W\,m^{-2}}$ and its luminosity is $6.2 \times 10^{32}\,\mathrm{W}$. Calculate the distance to the star in light years.

12 Two stars have the same size but one has a temperature that is four times larger.
   a Estimate how much more energy per second the hotter star radiates.
   b The apparent brightness of the two stars is the same; determine the ratio of the distance of the cooler star to that of the hotter star.

13 Two stars are the same distance from the Earth and their apparent brightnesses are $9.0 \times 10^{-12}\,\mathrm{W\,m^{-2}}$ (star A) and $3.0 \times 10^{-13}\,\mathrm{W\,m^{-2}}$ (star B). Calculate the ratio of the luminosity of star A to that of star B.

14 Take the surface temperature of our Sun to be 6000 K and its luminosity to be $3.9 \times 10^{26}\,\mathrm{W}$. Find, in terms of the solar radius, the radius of a star with:
   a temperature 4000 K and luminosity $5.2 \times 10^{28}\,\mathrm{W}$
   b temperature 9250 K and luminosity $4.7 \times 10^{27}\,\mathrm{W}$.

15 Two stars have the same luminosity. Star A has a surface temperature of 5000 K and star B a surface temperature of 10 000 K.
   a Suggest which is the larger star and by how much.
   b The apparent brightness of A is double that of B; calculate the ratio of the distance of A to that of B.

16 Star A has apparent brightness $8.0 \times 10^{-13}\,\mathrm{W\,m^{-2}}$ and its distance is 120 ly. Star B has apparent brightness $2.0 \times 10^{-15}\,\mathrm{W\,m^{-2}}$ and its distance is 150 ly. The two stars have the same size. Calculate the ratio of the temperature of star A to that of star B.

17 Calculate the apparent brightness of a star of luminosity $2.45 \times 10^{28}\,\mathrm{W}$ and a parallax of 0.034″.

Learning objectives

- Describe the use of stellar spectra.
- Work with the HR diagram, including representation of stellar evolution.
- Apply the mass–luminosity relation.
- Describe Cepheid variables and their use as standard candles.
- Describe the nature of the stars in the main regions of the HR diagram.
- Understand the limits on mass for white dwarfs and neutron stars.

D2 Stellar characteristics and stellar evolution

This section deals with the lives of stars on the main sequence and their evolution away from it. We will see how **stellar spectra** may be used to determine the chemical composition of stars, and will study an important diagram called the Hertzsprung–Russell (HR) diagram. We will follow the **stellar evolution** on the HR diagram and meet important classes of stars such as Cepheid variables, **white dwarfs** and **red giants**.

D2.1 Stellar spectra

The energy radiated by a star is in the form of electromagnetic radiation and is distributed over an infinite range of wavelengths. A star is assumed to radiate like a black body. Figure **D.9** shows black-body spectra at various temperatures.

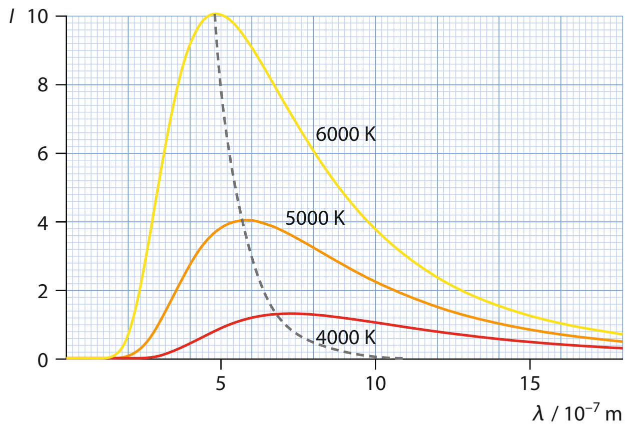


Figure D.9 Black-body radiation profiles at various temperatures. The broken lines show the variation with temperature of the peak intensity and the wavelength at which it occurs.

Much information can be determined about a star by examining its spectrum. The first piece of information is its surface temperature. Most of the energy is emitted around a wavelength called the peak wavelength. Calling this wavelength λ_0, we see that the colour of the star is mainly determined by the colour corresponding to λ_0.

The **Wien displacement law** relates the wavelength λ_0 to the surface temperature T:

$$\lambda_0 T = \text{constant} = 2.90 \times 10^{-3}\,\text{K m}$$

This implies that the higher the temperature, the lower the wavelength at which most of the energy is radiated.

Worked examples

D.9 The Sun has an approximate black-body spectrum with most of its energy radiated at a wavelength of 5.0×10^{-7} m. Find the surface temperature of the Sun.

From Wien's law, $5.0 \times 10^{-7}\,\text{m} \times T = 2.9 \times 10^{-3}\,\text{K m}$; that is, $T = 5800\,\text{K}$.

D.10 The Sun (radius $R = 7.0 \times 10^{8}$ m) radiates a total power of 3.9×10^{26} W. Find its surface temperature.

From $L = \sigma A T^4$ and $A = 4\pi R^2$, we find

$$T = \left(\frac{L}{\sigma 4\pi R^2}\right)^{1/4} \approx 5800\,\text{K}$$

The surface temperature of a star is determined by measuring the wavelength at which most of its radiation is emitted (see Figure **D.10**).

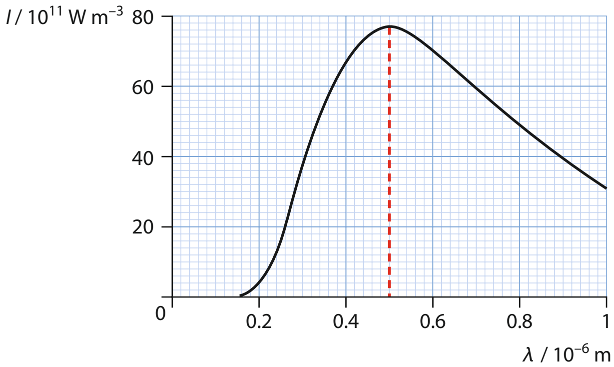


Figure D.10 The spectrum of this star shows a peak wavelength of 500 nm. Using Wien's law, we can determine its surface temperature.

The second important piece of information from a star's spectrum is its chemical composition. It is common to obtain an absorption spectrum in which dark lines are superimposed on a background of continuous colour (as in Figure **D.11**). Each dark line represents the absorption of light of a specific wavelength by a specific chemical element in the star's atmosphere.

Figure D.11 Absorption spectrum of a star, showing the absorption lines of hydrogen. A real spectrum would show very many dark lines corresponding to other elements as well.

It is known that most stars have essentially the same chemical composition, yet show different absorption spectra. The reason for this difference is that different stars have different temperatures. Consider two stars with the same content of hydrogen. One is hot, say at 25 000 K, and the other cool, say at 10 000 K. The hydrogen in the hot star is ionised, which means the electrons have left the hydrogen atoms. These atoms cannot absorb any light passing through them, since there are no bound electrons to absorb the photons and make transitions to higher-energy states. Thus, the hot star will not show any absorption lines at hydrogen wavelengths. The cooler star, however, has many of its hydrogen atoms in the energy state $n = 2$. Electrons in this state can absorb photons to make transitions to states such as $n = 3$ and $n = 4$, giving rise to characteristic hydrogen absorption lines. Similarly, an even cooler star – of temperature, say, 3000 K – will have most of the electrons in its hydrogen atoms in the ground state, so they can only absorb photons corresponding to ultraviolet wavelengths. These will not result in dark lines in an optical spectrum.

Stars are divided into seven **spectral classes** according to their colour (see Table **D.3**). As we have seen, colour is related to surface temperature. The spectral classes are called O, B, A, F, G, K and M (remembered as Oh Be A Fine Girl/Guy Kiss Me!).

Spectral class	Colour	Temperature / K
O	Electric blue	25 000–50 000
B	Blue	12 000–25 000
A	White	7 500–12 000
F	Yellow–white	6 000–7 500
G	Yellow	4 500–6 000
K	Orange	3 000–4 500
M	Red	2 000–3 000

Table D.3 Colour and temperature characteristics of spectral classes.

It is known from spectral studies that hydrogen is the predominant element in normal **main-sequence** stars, making up upto 70% of their mass, followed by helium with at 28%; the rest is made up of heavier elements.

Spectral studies also give information on the star's velocity and rotation (through the Doppler shifting of spectral lines) and the star's magnetic field owing to the splitting of spectral lines in a magnetic field.

D2.2 The Hertzsprung–Russell diagram

Astronomers realised early on that there was a correlation between the luminosity of a star and its surface temperature. In the early part of the twentieth century, the Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell independently pioneered plots of stellar luminosities. Such plots are now called **Hertzsprung–Russell** (HR) **diagrams**. In the HR diagram in Figure **D.12**, the vertical axis represents luminosity in units of the Sun's luminosity (that is, 1 on the vertical axis corresponds to the solar luminosity, $L_{\odot} = 3.9 \times 10^{26}$ W). The horizontal axis shows the surface temperature of the star (in kelvin). The temperature decreases to the right.

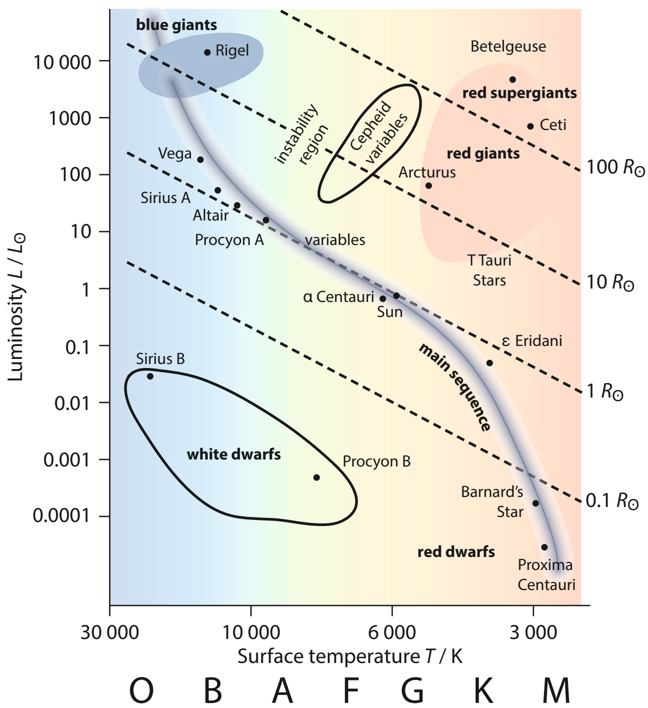


Figure D.12 A Hertzsprung–Russell diagram. Surface temperature increases to the left. Note that the scales are not linear.

Exam tip
It is very important that you clearly understand the HR diagram.

Also shown is the spectral class, which is an alternative way to label the horizontal axis. The luminosity in this diagram varies from 10^{-4} to 10^{4}, a full eight orders of magnitude, whereas the temperature varies from 3000 K to 30 000 K. For this reason, the scales on each axis are not linear.

The slanted dotted lines represent stars with the same radius, so our Sun and Procyon have about the same radius. The symbol $R_{\odot}$ stands for the radius of the Sun.

As more and more stars were placed on the HR diagram, it became clear that a pattern was emerging. The stars were not randomly distributed on the diagram. Three clear features emerge:

- Most stars fall on a strip extending diagonally across the diagram from top left to bottom right. This is called the **main sequence**.
- Some large stars, reddish in colour, occupy the top right. These are the **red giants** (large and cool). Above these are the **red supergiants** (very large and cool).
- The bottom left is a region of small stars known as **white dwarfs** (small and hot).

As we will see in Section **D2.3**, the higher the luminosity of a main-sequence star, the higher its mass. So as we move along the main sequence towards hotter stars, the masses of the stars increase. Thus, the right end of the main sequence is occupied by **red dwarfs** and the left by **blue giants**.

Note that, once we know the temperature of a star (for example, through its spectrum), the HR diagram can tell us its luminosity with an acceptable degree of accuracy, provided it is a main-sequence star.

Exam tip
Main-sequence stars: fuse hydrogen to form helium,
Red giants: bright, large, cool, reddish, tenuous.
Red supergiants: even larger and brighter than red giants,
White dwarfs: dim, small, hot, whitish, dense.

D2.3 Main-sequence stars

Our Sun is a typical member of the main sequence. It has a mass of 2×10^{30} kg, a radius of 7×10^{8} m and an average density of $1.4 \times 10^{3}\,\mathrm{kg\,m^{-3}}$, and it radiates at a rate of 3.9×10^{26} W. What distinguishes different main-sequence stars is their mass (see Figure **D.13**). Main-sequence stars produce enough energy in their core, from the nuclear fusion of hydrogen into helium, to exactly counterbalance the tendency of the star to collapse under its own weight. The common characteristic of all main-sequence stars is the fusion of hydrogen into helium.

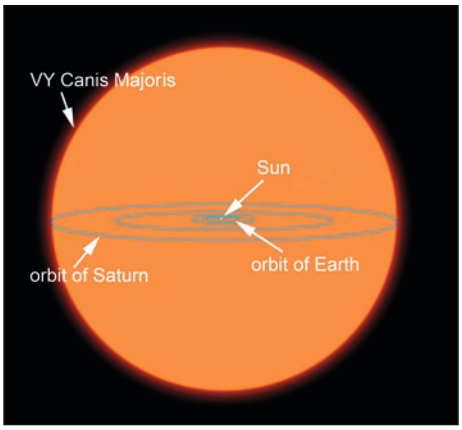


Figure D.13 Artist's impression of a comparison between our Sun and the red supergiant VY Canis Majoris.

D2.4 Red giants and red supergiants

Red giants are very large, cool stars with a reddish appearance. The luminosity of red giants is considerably greater than that of main-sequence stars of the same temperature. Treating them as black bodies radiating according to the Stefan–Boltzmann law means that a luminosity which is 10^{3} times greater than that of our Sun corresponds to a surface area which is 10^{3} times that of the Sun, and thus a radius about 30 times greater. The mass of a red giant can be as much as 100 times the mass of our Sun, but their huge size also implies small densities. A red giant will have a central hot core surrounded by an enormous envelope of extremely tenuous gas.

Red supergiants are even larger. Extreme examples include stars with radii that are 1500 times that of our Sun and luminosities of 5×10^{5} solar luminosities.

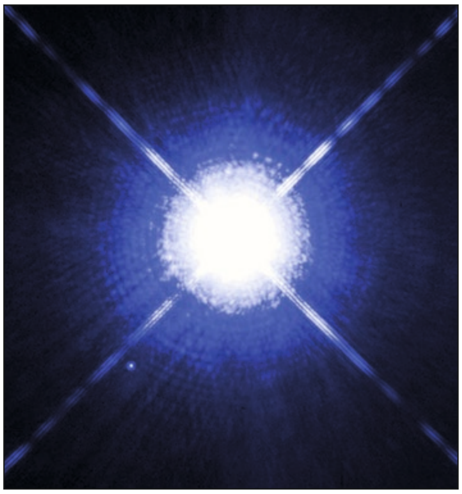


Figure D.14 Sirius A is the bright star in the middle of the photograph. Its white dwarf companion, Sirius B, is the tiny speck of light at the lower left. The rings and spikes are artefacts of the telescope's imaging system. The photograph has been overexposed so that the faint Sirius B can be seen.

D2.5 White dwarfs

White dwarf stars are common but their faintness makes them hard to detect. A well-known white dwarf is Sirius B, the second star in a binary star system (double star) whose other member, Sirius A, is the brightest star in the evening sky (Figure **D.14**).

Sirius A and Sirius B have about the same surface temperature (about 10 000 K) but the luminosity of Sirius B is about 10 000 times smaller. This means that it has a radius that is 100 times smaller than that of Sirius A. Here is a star with a mass roughly that of the Sun with a size similar to that of the Earth. This means that its density is about 10^6 times the density of the Earth!

Worked example

D.11 A main-sequence star emits most of its energy at a wavelength of 2.4×10^{-7} m. Its apparent brightness is measured to be $4.3 \times 10^{-9}\,\mathrm{W\,m^{-2}}$. Estimate the distance of the star.

From Wien's law, we find the temperature of the star to be given by

$$\lambda_0 T = 2.9 \times 10^{-3}\,\mathrm{K\,m}$$

$$\Rightarrow T = \frac{2.9 \times 10^{-3}}{2.4 \times 10^{-7}}\,\mathrm{K}$$

$$= 12\,000\,\mathrm{K}$$

From the HR diagram in Figure **D.12**, we see that such a temperature corresponds to a luminosity about 100 times that of the Sun: that is, $L = 3.9 \times 10^{28}$ W. Thus,

$$d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3.9 \times 10^{28}}{4\pi \times 4.3 \times 10^{-9}}}\,\mathrm{m}$$

$$= 8.5 \times 10^{17}\,\mathrm{m} \approx 90\,\mathrm{ly} \approx 28\,\mathrm{pc}$$

D2.6 The mass–luminosity relation

For stars on the main sequence, there exists a relation between the mass and the luminosity of the star. The **mass–luminosity relation** states that

$$L \propto M^{3.5}$$

This relation comes from application of the laws of nuclear physics to stars. Main-sequence stars in the upper left-hand corner of the HR diagram have a very high luminosity and therefore are very massive.

Exam tip
The mass–luminosity relation can only be used for main-sequence stars.

Worked example

D.12 Use the HR diagram and the mass–luminosity relation to estimate the ratio of the density of Altair to that of the Sun.

The two stars have the same radius and hence the same volume. The luminosity of Altair is about 10 times that of the Sun. From

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

we get

$$10 = \left(\frac{M}{M_\odot}\right)^{3.5} \quad \Rightarrow \quad \frac{M}{M_\odot} = 10^{1/3.5} \approx 1.9$$

Hence the ratio of densities is also about 1.9, since the volumes are the same.

D2.7 Cepheid stars

Cepheid variable stars are stars whose luminosity is not constant in time but varies **periodically** from a minimum to a maximum, the periods being typically from a couple of days to a couple of months. The brightness of the star increases sharply and then fades off more gradually, as shown in the **light curve** of a Cepheid in Figure **D.15**.

The mechanism for the periodic variation of the luminosity of Cepheid stars is the following. As radiation rushes outwards it ionises helium atoms in the atmosphere of the star. The freed electrons, through collisions, heat up the star's atmosphere. This increases the pressure, which forces the outer layers of the star to expand. When most of the helium is ionised, radiation now manages to leave the star, and the star cools down and begins to contract under its own weight. This makes helium nuclei recombine with electrons, and so the cycle repeats as helium can again be ionised. The star is brightest when the surface is expanding outwards at maximum speed.

Cepheids occupy a strip between the main sequence and the red giants on an HR diagram.

At the beginning of the 20th century, astronomer Henrietta Leavitt discovered a remarkably precise relationship between the average luminosity of Cepheids and their period. The longer the period, the larger the luminosity (see Figure D.16). This makes Cepheid stars **standard candles** – that is, stars of a known luminosity, obtained by measuring their period.

Exam tip
The reason for a Cepheid star's periodic variation in luminosity is the periodic expansion and contraction of the outer layers of the star.

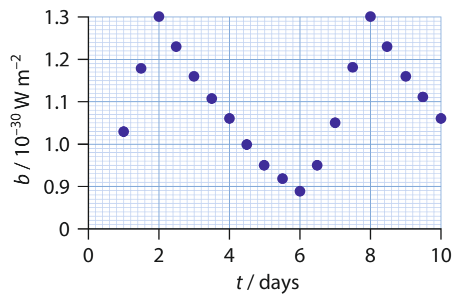


Figure D.15 The apparent brightness of a Cepheid star varies periodically with time.

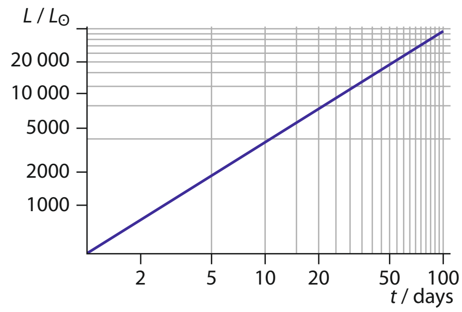


Figure D.16 The relationship between peak luminosity and period for Cepheid stars.

Worked example

D.13 Estimate the distance of the Cepheid whose light curve is shown in Figure **D.15**.

The period is 6 days. From Figure **D.16**, this corresponds to a luminosity of about 2000 solar luminosities, or about $L = 7.2 \times 10^{29}$ W. The average apparent brightness is $b = 1.1 \times 10^{-10}\,\mathrm{W\,m^{-2}}$. Therefore

$$b = \frac{L}{4\pi d^2}$$

$$\Rightarrow d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{7.2 \times 10^{29}}{4\pi \times 1.1 \times 10^{-10}}}\ \mathrm{m}$$

$$= 2.28 \times 10^{19}\,\mathrm{m} \approx 2400\,\mathrm{ly} \approx 740\,\mathrm{pc}$$

Thus, one can determine the distance to the galaxy in which a Cepheid is assumed to be. The Cepheid method can be used to find distances up to a few megaparsecs.

D2.8 Stellar evolution: the Chandrasekhar and Oppenheimer–Volkoff limits

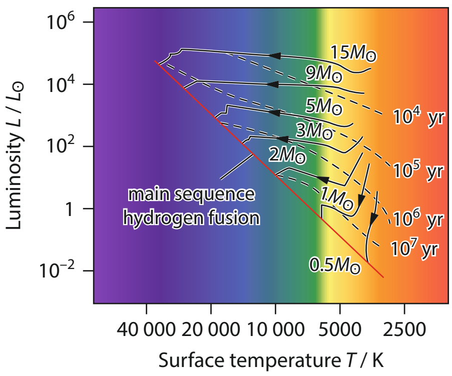


Figure D.17 Evolutionary tracks of protostars as they approach the main sequence. $M_\odot$ stands for one solar mass.

Stars are formed out of contracting gases and dust in the **interstellar medium**, which has hydrogen as its main constituent. Initially the star has a low surface temperature and so its position is somewhere to the right of the main sequence on the HR diagram. As the star contracts under its own weight, gravitational potential energy is converted into thermal energy and the star heats up; it begins to move towards the main sequence. The time taken to reach the main sequence depends on the mass of the star; heavier stars take less time. Our Sun, an average star, has taken about 20 to 30 million years (see Figure **D.17**).

As a star is compressed more and more (under the action of gravity), its temperature rises and so does its pressure. Eventually, the temperature in the core reaches 5×10^6 to 10^7 K and nuclear fusion reactions commence, resulting in the release of enormous amounts of energy. The energy released can account for the sustained luminosity of stars such as our Sun, for example, over the 4–5 billion years of its life so far. Thus, nuclear fusion provides the energy that is needed to keep the star hot, so that its pressure is high enough to oppose further contraction, and at the same time to provide the energy that the star is radiating into space.

On the main sequence, the main nuclear fusion reactions are those of the proton–proton cycle (Section **D1.2**), in which the net effect is to turn four hydrogen nuclei into one helium-4 nucleus.

When about 12% of the hydrogen in the star has been used up in nuclear fusion, a series of instabilities develops in the star, upsetting the delicate balance between radiation pressure and gravitational pressure. The star will then begin to move away from the main sequence. What happens next is determined mainly by the mass of the star. Other types of nuclear fusion reactions will take place (see Section **D4**) and the star will change in size and surface temperature (and hence colour).

The mass of a star is the main factor that determines its evolution off the main sequence.

The changes that take place can be shown as paths on the HR diagram. We may distinguish two essentially different paths, the first for what we will call **low-mass** stars, with a mass less than about eight solar masses, and the second for stars with a higher mass. (In reality the situation is more complex, but this simple distinction is sufficient for the purposes of this course.)

In low-mass stars, helium collects in the core of the star, surrounded by a thin shell of hydrogen and a bigger hydrogen envelope (Figure **D.18**).

Only hydrogen in the thin inner shell undergoes nuclear fusion to helium. The temperature and pressure of the helium build up and eventually helium itself begins to fuse (this is called the 'helium flash'), with helium in a thin inner shell producing carbon in the core. In the core, some carbon nuclei fuse with helium to form oxygen. Oxygen is the heaviest element that can be produced in low-mass stars; the temperature never rises enough for production of heavier elements. The hydrogen in the thin shell is still fusing, so the star now has nuclear fusion in two shells, the H and He shells. The huge release of energy blows away the outer layers of the star in an explosion called a **planetary nebula**; mass is thrown into space, leaving behind the carbon core (and some oxygen). This evolution may be shown on an HR diagram (Figure **D.19**). We will return to the processes in the core in Section **D4.2**.

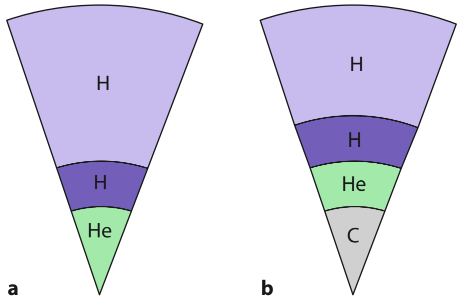


Figure D.18 **a** The structure of a low-mass star after it leaves the main sequence. **b** After helium begins to fuse, carbon collects in the core.

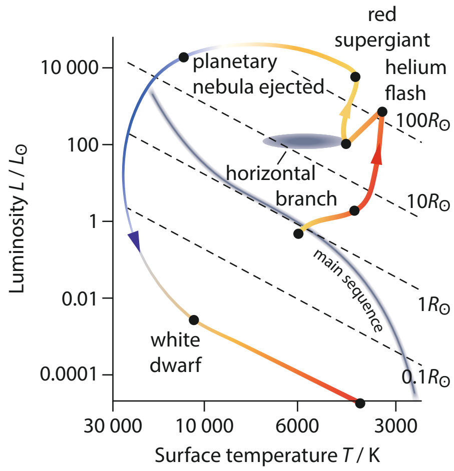


Figure D.19 Evolutionary path of a low-mass star. This is the path of a star of one solar mass that ends up as a white dwarf, which continues to cool down, moving the star ever more to the right on the HR diagram.

The path takes the star off the main sequence and into the red giant region. The star gets bigger and cooler on the surface and hence becomes red in colour. The time taken to leave the main sequence and reach the planetary nebula stage is short compared with the time spent on the main sequence: it takes from a few tens to a few hundreds of million of years. The path then takes the star to the white dwarf region. The star is now a stable but dead star (Figure **D.20**). No nuclear reactions take place in the core.

The conditions in the core mean that the electrons behave as a gas, and the pressure they generate is what keeps the core from collapsing further under its weight. This pressure is called **electron degeneracy pressure** and is the result of a quantum mechanical effect, referred to as the Pauli Exclusion Principle, which states that no two electrons may occupy the same quantum state.

The core has now become a **white dwarf** star. Now exposed, and with no further energy source, the star is doomed to cool down to practically zero temperature and will then become a **black dwarf**.

Figure D.20 The Helix, a planetary nebula. The star that produced this nebula can be seen at its exact centre.

Electron degeneracy pressure prevents the further collapse of the core and – provided the mass of the core is less than about 1.4 solar masses – the star will become a stable white dwarf. This important number is known in astrophysics as the **Chandrasekhar limit**.

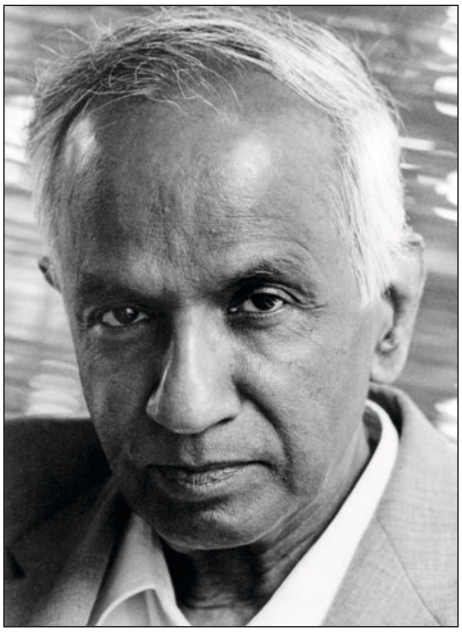


Figure D.21 Subrahmanyan Chandrasekhar (1910–1995).

The limit is named after the astrophysicist Subrahmanyan Chandrasekhar (Figure **D.21**), who discovered it in the 1930s.

We now look at the evolution of stars whose mass is greater than about eight solar masses. The process begins much the same way as it did for low-mass stars, but differences begin to show when carbon fuses with helium in the core to form oxygen. If the mass of the star is large enough, the pressure caused by gravity is enough to raise the temperature sufficiently to allow the formation of ever-heavier elements: neon, more oxygen, magnesium and then silicon; eventually iron is produced in the most massive stars, and that is where the process stops, since iron is near the peak of the binding-energy curve. It would require additional energy to be supplied for iron to fuse.

The star moves off the main sequence and into the red supergiant area (Figure **D.22**). As the path moves to the right, ever-heavier elements are produced. The star is very hot in the core. Photons have enough energy at these temperatures to split nuclei apart; in about one second (!) millions of years, worth of nuclear fusion is undone. Nuclei are in turn ripped apart into individual protons and neutrons, so that in a very short time the star is composed mainly of protons, electrons, neutrons and photons.

Exam tip
Fusion ends with the production of iron.

Because of the high densities involved, the electrons are forced into the protons, turning them into neutrons and producing neutrinos that escape from the star ($e^- + p \rightarrow n + \nu_e$). The star's core is now made up almost entirely of neutrons, and is still contracting rapidly. The Pauli Exclusion Principle may now be applied to the neutrons: if they get too close to one another, a pressure develops to prevent them from getting any closer. But they have already done so, and so the entire core now rebounds to a larger equilibrium size. This rebound is catastrophic for the star, creating an enormous shock wave travelling outwards that tears the outer layers of the star apart. The resulting explosion, called a **supernova**, is much more violent than a planetary nebula. The energy loss from this explosion leads to a drastic drop in the temperature of the star, and it begins to collapse.

Exam tip
This summary and the paths on the HR diagram are the bare minimum you should know for an exam.

The core that is left behind, which is more massive than the Chandrasekhar limit, will most likely become a **neutron star**. Neutron pressure keeps such a star stable, provided the mass of the core is not more than about 2–3 solar masses – the **Oppenheimer–Volkoff limit**. If its mass higher than this, it may collapse further and become a black hole.

Table **D.4** shows the temperatures at which various elements participate in fusion reactions.

Element	$T / 10^6$ K	Where
Hydrogen	1–20	Main sequence
Helium	100	Red giant
Carbon	500–800	Supergiant
Oxygen	1000	Supergiant

Table D.4 Temperatures at which various elements participate in fusion reactions.

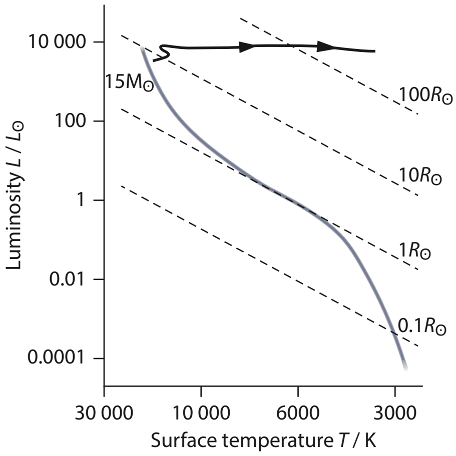


Figure D.22 Evolutionary path of a star of 15 solar masses. It becomes a red supergiant that explodes in a supernova. After the supernova, the star becomes a neutron star, whose luminosity is too small to be plotted on the HR diagram.

To summarise:

If the mass of the core of a star is less than the Chandrasekhar limit of about 1.4 solar masses, it will become a stable white dwarf, in which electron pressure keeps the star from collapsing further.

If the core is more massive than the Chandrasekhar limit but less than the Oppenheimer–Volkoff limit of about 2–3 solar masses, the core will collapse further until electrons are driven into protons, forming neutrons. Neutron pressure now keeps the star from collapsing further, and the star becomes a neutron star.

If the Oppenheimer–Volkoff limit is exceeded, the star will become a black hole.

Initial mass of star (in terms of solar masses)	Outcome
0.08–0.25	White dwarf with helium core
0.25–8	White dwarf with carbon core
8–12	White dwarf with oxygen/neon/magnesium core
12–40	Neutron star
>40	Black hole

Table D.5 The final fate of stars with various initial masses.

Table **D.5** shows the final end products of evolution for different initial stellar masses. Figure **D.23** is a schematic summary of the life history of a star.

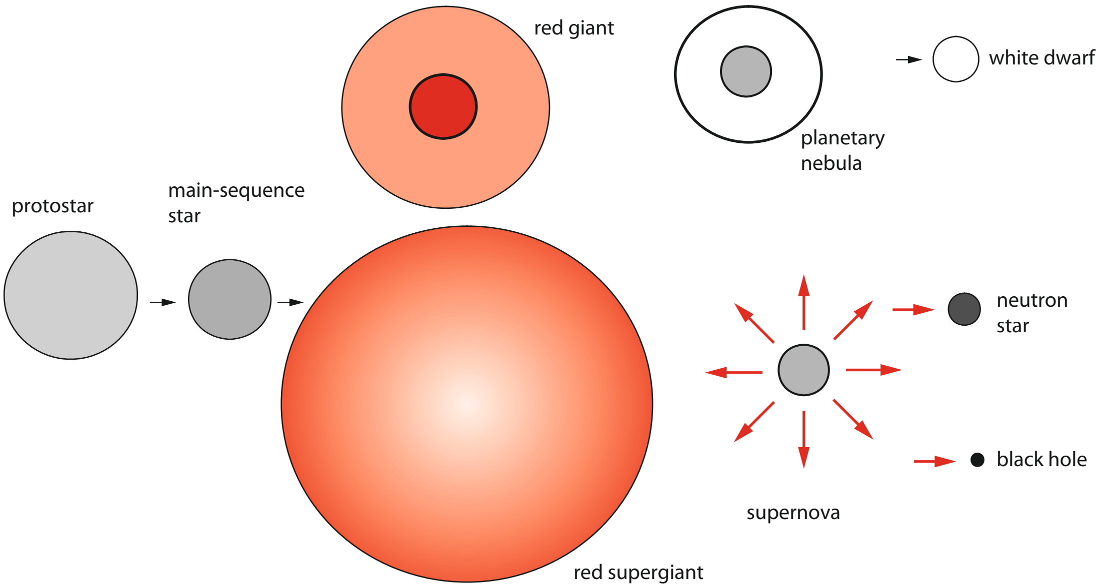


Figure D.23 The birth and death of a star. The star begins as a protostar, evolves to the main sequence and then becomes a red giant or supergiant. After a planetary nebula or supernova explosion, the core of the star develops into one of the three final stages of stellar evolution: white dwarf, neutron star or black hole.

Nature of science

Evidence from starlight

The light from a star is the best source of information about it. The distribution of frequencies tells us its surface temperature, and the actual frequencies present tell us its composition, as each element has a characteristic spectrum. The luminosity and temperature of a star are related, and together give us information about the evolution of stars of different masses. Using this evidence, Chandrasekhar predicted a limit to the mass of a star that would become a white dwarf, while Oppenheimer and Volkoff predicted the mass above which it would become a black hole. The development of theories of stellar evolution illustrates how, starting from simple observations of the natural world, science can build up a detailed picture of how the universe works. Further observations are then needed to confirm or reject hypotheses.

Test yourself

18 Describe how a stellar absorption spectrum is formed.

19 Describe how the chemical composition of a star may be determined.

20 Describe how the colour of the light from a star can be used to determine its surface temperature.

21 Stars A and B emit most of their light at wavelengths of 650 nm and 480 nm, respectively. Star A has twice the radius of star B. Find the ratio of the luminosities of the stars.

22 **a** State what is meant by a Hertzsprung–Russell (HR) diagram.

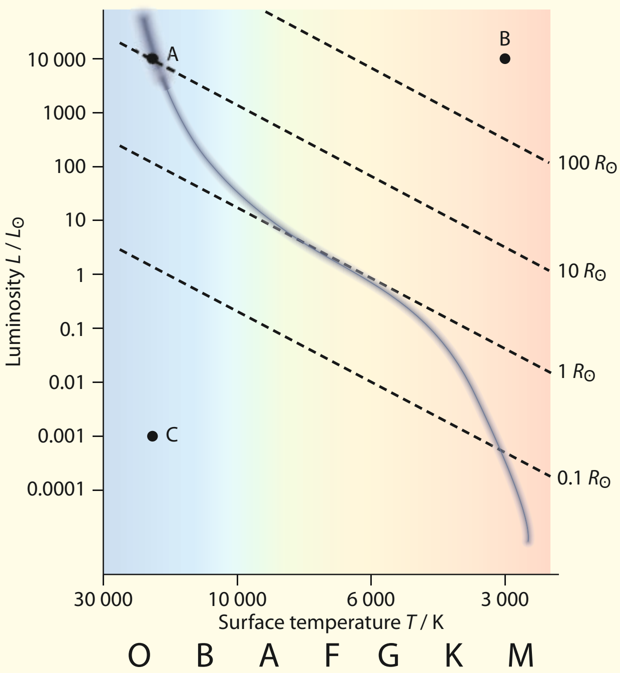
b Describe the main features of the HR diagram.

c The luminosity of the Sun is 3.9×10^{26} W and its radius is 7.0×108 m. For star A in the HR diagram below **calculate**

i the temperature

ii the density in terms of the Sun's density.

d For stars B and C calculate the radius in terms of the Sun's radius.



23 A main-sequence star emits most of its energy at a wavelength of 2.42×10^{-7} m. Its apparent brightness is measured to be 8.56×10^{-12} W m^{-2}. Estimate its distance using the HR diagram in question **22**.

24 A main-sequence star is 15 times more massive than our Sun. Calculate the luminosity of this star in terms of the solar luminosity.

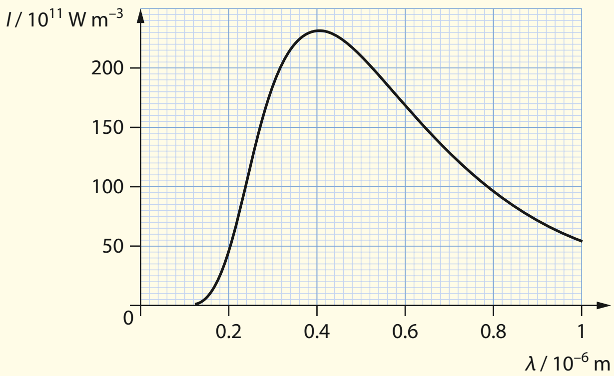
25 **a** The luminosity of a main-sequence star is 4500 times greater than the luminosity of our Sun. Estimate the mass of this star in terms of the solar mass.

b A star has a mass of 12 solar masses and a luminosity of 3200 solar luminosities. Determine whether this could be a main-sequence star.

26 Describe the mechanism by which the luminosity of Cepheid stars varies.

27 Using Figure **D.16**, calculate the distance of a Cepheid variable star whose period is 10 days and whose average apparent brightness is 3.45×10^{-14} W m^{-2}.

28 **a** Find the temperature of a star whose spectrum is shown below.



b Assuming this is a main-sequence star, *estimate* its luminosity using the HR diagram in question **22**.

29 Estimate the temperature of the universe when the peak wavelength of the radiation in the universe was 7.0×10^{-7} m.

30 A neutron star has a radius of 30 km and makes 500 revolutions per second.

a Calculate the speed of a point on its equator.

b Determine what fraction of the speed of light this is.

31 Describe the formation of a red giant star.

32 **a** Describe what is meant by a **planetary nebula**.

b Suggest why most photographs show planetary nebulae as rings: doesn't the gas surround the core in all directions?

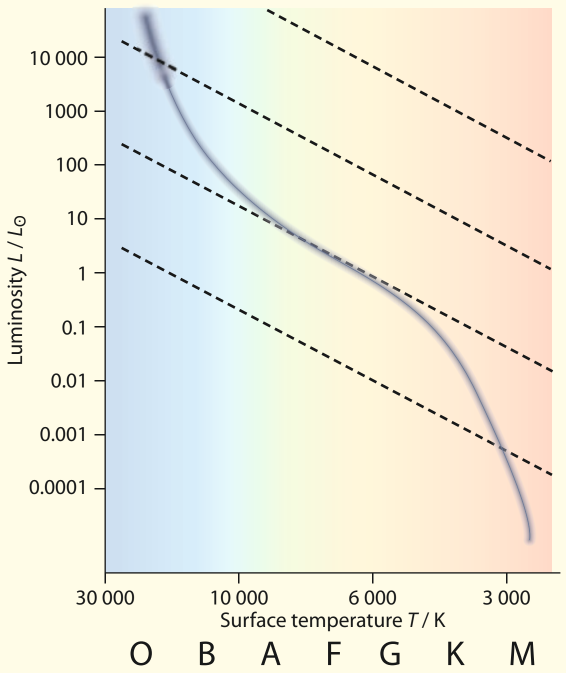
33 Assume that no stars of mass greater than about two solar masses could form anywhere. Would life as we know it on Earth be possible?

34 Describe the evolution of a main-sequence star of mass:

a 2 solar masses

b 20 solar masses.

c Show the evolutionary paths of these stars on a copy of the HR diagram below.



35 **a** Describe the formation of a white dwarf star.

b List two properties of a white dwarf.

c Describe the mechanism which prevents a white dwarf from collapsing under the action of gravity.

36 Describe **two** differences between a main-sequence star and a white dwarf.

37 A white dwarf, of mass half that of the Sun and radius equal to one Earth radius, is formed. Estimate its density.

38 Describe **two** differences between a main-sequence star and a neutron star.

39 **a** Describe the formation of a neutron star.

b List two properties of a neutron star.

c Describe the mechanism which prevents a neutron star from collapsing under the action of gravity.

40 Describe your understanding of the **Chandrasekhar limit**.

41 Describe your understanding of the **Oppenheimer–Volkoff limit**.

42 Assume that the material of a main-sequence star obeys the ideal gas law, $PV = NkT$. The volume of the star is proportional to the cube of its radius R, and N is proportional to the mass M of the star.

a Show that $PR^3 \propto MT$.

The star is in equilibrium under the action of its own gravity, which tends to collapse it, and the pressure created by the outflow of energy from its interior, which tends to expand it. It can be shown that this equilibrium results in the condition $P \propto \frac{M^2}{R^2}$. **(Can you see how?)**

b Combine these two proportionalities to show that $P \propto \frac{M}{R}$. Use this result to explain that, as a star shrinks, its temperature goes up.

c Conclude this rough analysis by showing that the luminosity of main-sequence stars of the same density is given by $L \propto M^{3.3}$.

Learning objectives

- Understand Hubble's law.
- Understand the scale factor and red-shift.
- Understand the cosmic microwave background radiation.
- Understand the accelerating universe and red-shift.

D3 Cosmology

This section deals with three dramatic discoveries in cosmology: the discovery of the expansion of the universe by Hubble, the discovery of the **cosmic background radiation** by Penzias and Wilson, and finally the discovery of the accelerated rate of expansion by Perlmutter, Schmidt and Riess.

D3.1 Hubble's law: the expanding universe

Early in the 20th century, studies of galaxies revealed red-shifted absorption lines. Application of the standard Doppler effect indicated that these galaxies were moving **away from us**. By 1925, 45 galaxies had been studied, and all but the closest ones appeared to be moving away at enormous speeds.

Physics in distant galaxies is the same as that on Earth

How do we know the wavelength of light emitted by distant galaxies? Light emitted from galaxies comes from atomic transitions in the hot gas in the interior of the galaxies, which is mostly hydrogen. Galaxies are surrounded by cooler gas and thus light travelling through is absorbed at specific wavelengths, showing a characteristic absorption spectrum. The wavelengths corresponding to the dark lines are well known from experiments on Earth.

The velocity of recession is found by an application of the Doppler effect to light. Light from galaxies arrives on Earth red-shifted. This means that the wavelength of the light measured upon arrival is longer than the wavelength at emission. According to the Doppler effect, this implies that the source of the light – the galaxy – is moving away from observers on Earth.

The Doppler effect *may* be used to describe the red-shift in the light from distant galaxies. However, the red-shift is a consequence of the expanding universe in the sense that the space between galaxies is stretching out (expands) and this gives the illusion of galaxies moving away from each other; see Section **D3.2**.

If λ_0 is the wavelength of a spectral line and λ is the (longer) wavelength received on Earth, the red-shift z of the galaxy is defined as

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

If the speed v of the receding galaxy is small compared with the speed of light c, then the Doppler formula is $z = \frac{v}{c}$, which shows that the red-shift is indeed directly proportional to the receding galaxy's speed (more correctly, the component of its velocity along the line of sight).

In 1925, Edwin Hubble began a study to measure the distance to the galaxies for which the velocities of recession had been determined. In

Exam tip

Notice that $z = \frac{\lambda - \lambda_0}{\lambda_0}$ can also be written as $z = \frac{\lambda}{\lambda_0} - 1$.

The proper interpretation of the red-shift is not through the Doppler effect (even though we use the Doppler formula) but the stretching of space in between galaxies as the universe expands.

1929, Hubble announced that distant galaxies move away from us with speeds that are proportional to their distance (Figure **D.24**).

Hubble studied a large number of galaxies and found that the more distant the galaxy, the faster it moves away from us. This is **Hubble's law**, which states that the velocity of recession is directly proportional to the distance, or

$$v = H_0 d$$

where d is the distance between the Earth and the galaxy and v is the galaxy's velocity of recession. The constant of proportionality, H_0, is the slope of the graph and is known as the **Hubble constant**.

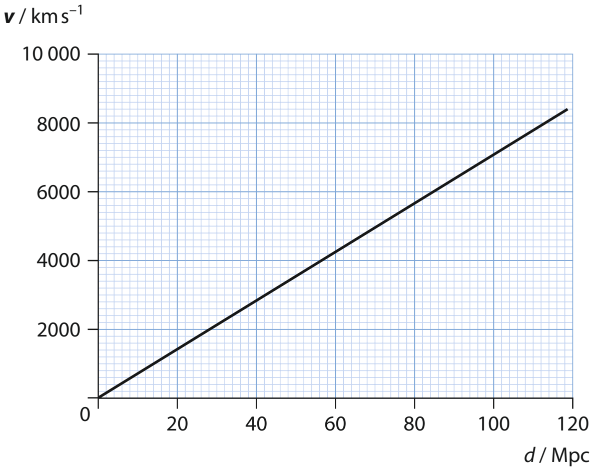


Figure D.24 Hubble discovered that the velocity of recession of galaxies is proportional to their distance from us.

Using $z = \frac{v}{c}$, we can rewrite Hubble's law as

$$z = \frac{H_0 d}{c} \quad \Rightarrow \quad d = \frac{cz}{H_0}$$

This formula relates distance to red-shift. It is an approximate relation, valid only for values of the red-shift z up to about 0.2.

There has been considerable debate as to the value of the Hubble constant. The most recent value, provided by the ESA's Planck satellite observatory data, is $H_0 = 67.80 \pm 0.77\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Worked example

D.14 A hydrogen line has a wavelength of 434 nm. When received from a distant galaxy, this line is measured on Earth to be at 486 nm. Calculate the speed of recession of this galaxy.

The red-shift is $\frac{486-434}{434} = 0.12$, so $v = 0.12c = 3.6 \times 10^7\,\mathrm{m\,s^{-1}}$.

Hubble's discovery implies that, in the past, the distances between galaxies were smaller and, moreover, that at a specific time in the past the entire universe had the size of a point. This specific time is taken to be the beginning of the universe, and leads to the model of the universe known as the **Big Bang model**, to be discussed in Section **D3.3**. Not only time, but also the space in which the matter and energy of the universe reside were created at that moment. As the space expanded, the distance between clumps of matter increased, leading to the receding galaxies that Hubble observed.

Hubble's law does not imply that the Earth is at the centre of the universe, even though the observation of galaxies moving away from us might lead us to believe so. An observer on a different star in a different galaxy would reach the same (erroneous) conclusion about their location.

D3.2 The cosmic scale factor *R* and red-shift

The expansion of the universe can be described in terms of a scale factor, R. To understand what this is, consider two points with coordinates 1 and 3 on a number line (Figure **D.25a**). In ordinary geometry we would have no problem saying that the distance between the two points is the difference in their coordinates, $3-1=2$ units. Let us call this difference in coordinates Δx. If space expands, however, after some time the diagram would look like Figure **D.25b**. The distance between the points has increased but the difference in their coordinates has remained the same. So this difference does not give the actual physical distance between the points.

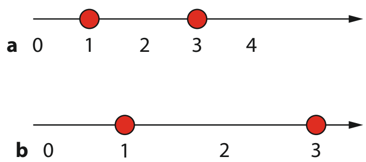


Figure D.25 As the space between two points stretches, the physical distance between them increases, even though the coordinates of the points do not change.

The meaning of the scale factor R is that multiplying the difference in coordinates Δx by R gives the physical distance d between the points:

$$d = R\Delta x$$

The scale factor may depend on time. The function $R(t)$ is called the **scale factor** of the universe and is of basic importance to cosmology. It is sometimes referred to (very loosely) as the **radius of the universe**.

This gives a new and completely different interpretation of red-shift. Suppose that, when a photon of cosmic microwave background (CMB) radiation was emitted in the very distant past, its wavelength was λ_0. Let Δx stand for the difference in the coordinates of two consecutive wave crests. Then

$$\lambda_0 = R_0\Delta x$$

where R_0 is the value of the scale factor at the time of emission. If this same wavelength is observed now, its value will be

$$\lambda = R\Delta x$$

where R is the value of the scale factor at the present time. We deduce that

$$\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$$

Thus the explanation for the observed red-shift in the light received from distant galaxies is not that the galaxies are really moving but that the space in between us and the galaxies is stretching (i.e. expanding).

Exam tip
SL students should know the result $z = \frac{R}{R_0} - 1$ but will not be examined on its derivation. HL students must know its derivation.

Of course, the stretching of space does give the illusion of motion, which is why applying the Doppler formula also gives the red-shift. But it must be stressed that the real explanation of the red-shift is the expansion of space and not the Doppler effect (see Figure **D.26**).

So the red-shift formula $z = \frac{\lambda - \lambda_0}{\lambda_0}$ becomes $z = \frac{R}{R_0} - 1$.

One of the great problems in cosmology is to determine how the scale factor depends on time. We will look at this problem in Section **D5.2**.

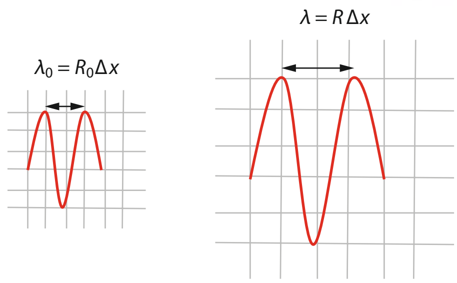


Figure D.26 As the universe expands, the wavelength of a photon emitted in the distant past increases when measured at the present time.

Worked examples

D.15 The peak wavelength of the CMB radiation at present is about 1 mm. In the past there was a time when the peak wavelength corresponded to blue light (400 nm). Estimate the size of the universe then, compared with its present size.

From $\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$ we find

$$\frac{R}{R_0} = \frac{1 \times 10^{-3}}{4 \times 10^{-7}} \approx 2 \times 10^3$$

So when the universe was bathed in blue light it was smaller by a factor of about 2000.

D.16 Determine the size of the universe relative to its present size at the time when the light of Worked example **D.14** was emitted.

The red-shift was calculated to be $z = 0.12$, so from $z = \frac{R}{R_0} - 1$ we find

$$0.12 = \frac{R}{R_0} - 1 \quad \Rightarrow \quad \frac{R}{R_0} = 1.12$$

The universe was 1.12 times smaller or $\frac{100\%}{1.12} = 89\%$ of its present size when the light was emitted.

D3.3 The Hot Big Bang model: the creation of space and time

The discovery of the expanding universe by Hubble implies a definite beginning, some 13.8 billion years ago. The size of the universe at that time was infinitesimally small and the temperature was enormous. Time, space, energy and mass were created at that instant. It is estimated that just 10^{-44} s after the beginning (the closest to $t = 0$ about which something remotely reliable may be said), the temperature was of order 10^{32} K. These conditions create the picture of a gigantic explosion at $t = 0$, which set matter moving outwards. Billions of years later, we see the remnants of this explosion in the receding motion of the distant galaxies. This is known as the **Hot Big Bang** scenario in cosmology.

The Big Bang was not an explosion that took place at a specific time in the past somewhere in the universe. At the time of the Big Bang, the

Why is the night sky dark?

The astronomers de Cheseaux and Olbers asked the very simple question of why the night sky is dark. Their argument, based on the prevailing static and infinite cosmology of the period, led to a night sky that would be uniformly bright! In its simplest form, the argument says that no matter where you look you will end up with a star. Hence the night sky should be uniformly bright, which it is not. This is Olbers' Paradox.

space in which the matter of the universe resides was created as well. Thus, the Big Bang happened about 13.8 billion years ago everywhere in the universe (the universe then being a point).

It is important to understand that the universe is not expanding into empty space. The expansion of the universe is not supposed to be like an expanding cloud of smoke that fills more and more volume in a room. The galaxies that are moving away from us are not moving into another, previously unoccupied, part of the universe. *Space is being stretched* in between the galaxies and so the distance between them is increasing, creating the illusion of motion of one galaxy relative to another.

There is plenty of experimental evidence in support of the Big Bang model. The first is the observation of an expanding universe that we have already talked about. The next piece of evidence is the cosmic microwave background radiation, to be discussed in Section **D3.4**.

If we assume that the expansion of the universe has been constant up to now, then $\frac{1}{H_0}$ gives an upper bound on the **age of the universe**. This is only an upper bound, since the fact that expansion rate was faster at the beginning implies a younger universe. The time $\frac{1}{H_0}$, known as the **Hubble time**, is about 14 billion years. The universe cannot be older than that. A more detailed argument that leads to this conclusion is as follows. Imagine a galaxy which is now at a distance d from us. Its velocity is thus $v=H_0d$. In the beginning the galaxy and the Earth were at zero separation from each other. If the present separation of d was thus covered at the same constant velocity H_0d, the time T taken to achieve this separation must be given by $H_0d=\frac{d}{T}$, that is, $T=\frac{1}{H_0}$. T is thus a measure of the age of the universe.

Exam tip
The inverse of the Hubble constant gives an **upper bound** on the age of the universe – that is, the actual age is less. This is because the estimate is based on a constant rate of expansion equal to the present rate.

The numerical value of the Hubble time with $H_0=67.80\times10^3\,\mathrm{m\,s^{-1}\,Mpc^{-1}}$ is

$$T_{\mathrm{H}}=\frac{1}{H}$$

$$=\frac{1}{67.80\times10^3\,\mathrm{m\,s^{-1}\,Mpc^{-1}}}=\frac{1}{67.80\times10^3\,\mathrm{m\,s^{-1}}}\times10^6\,\mathrm{pc}$$

$$=\frac{1}{67.80\times10^3\,\mathrm{m\,s^{-1}}}\times10^6\times3.09\times10^{16}\,\mathrm{m}=4.557\times10^{17}\,\mathrm{s}$$

$$=\frac{4.557\times10^{17}\,\mathrm{s}}{365\times24\times60\times60\,\mathrm{s\,yr^{-1}}}=14.5\times10^9\,\mathrm{yr}$$

This assumes that the universe has been expanding at a constant rate. This is not the case, and so this is an overestimate. The actual age of the universe according to the data from the Planck satellite is 13.8 billion years.

Exam tip
The characteristics of the cosmic microwave background are:

- a spectrum corresponding to black-body radiation at a temperature of 2.7 K.
- peak radiation in the microwave region.
- isotropic radiation with no apparent source.

D3.4 The cosmic microwave background radiation

In 1964, Arno Penzias and Robert Wilson, two radio astronomers working at Bell Laboratories, made a fundamental, if accidental, discovery. They were using an antenna they had just designed to study radio signals from our galaxy. But the antenna was picking up a signal that persisted no matter what part of the sky the antenna was pointing at. The spectrum of this signal (that is, the amount of energy as a function of the wavelength) turned out to be a black-body spectrum corresponding to a temperature of 2.7 K. The **isotropy** of this radiation (the fact that it was the same in all directions) indicated that it was not coming from any particular spot in the sky; rather, it was radiation that was filling all space.

Penzias and Wilson did not know that this kind of radiation had been predicted on the basis of the Big Bang theory 30 years earlier by George Gamow and his co-workers, and more recently by Jim Peebles and Robert Dicke at Princeton. The Princeton group was in fact planning to start a search for this radiation when the news of the discovery arrived.

Penzias and Wilson, with help from the Princeton group, realised that the radiation detected was the remnant of the hot explosion at the beginning of time. It was the afterglow of the enormous temperatures that existed in the very early universe. As the universe has expanded, the temperature has fallen to its present value of 2.7 K.

Since the work of Penzias and Wilson, a number of satellite observatories – COBE (COsmic Background Explorer), WMAP (Wilkinson Microwave Anisotropy Probe) and the Planck satellite – have verified the black-body nature of this cosmic microwave background radiation to extraordinary precision and measured its present temperature to be 2.723 K.

The COBE collaboration was headed by John Mather, who was in charge of over a thousand scientists and engineers.

Worked example

D.17 Find the wavelength at which most CMB radiation is emitted.

From the Wien displacement law, $\lambda T = 2.9 \times 10^{-3}$ K m, it follows that most of the energy is emitted at a wavelength of $\lambda = 1.07$ mm, which is in the microwave region.

D3.5 The accelerating universe and red-shift

It was expected that the rate of expansion of the universe should be slowing down. This was a reasonable expectation based on the fact that gravity should be pulling back on the distant galaxies, slowing them down. Cosmologists were very much interested in determining the value of the **deceleration parameter** of the universe, a dimensionless number called q_0 that would quantify the deceleration. A positive value of q_0 would indicate deceleration and a slowing down of the expansion rate. No one doubted that q_0 would be positive; the only issue was its actual value.

Two groups started work in this direction. The first group, under Saul Perlmutter, started the search in 1988 and the other, led by Brian Schmidt and Adam Riess, started in 1994.

We mentioned earlier that the distance–red-shift relation, $d = \dfrac{cz}{H_0}$, is only approximate, valid for $z < 0.2$. But the two groups were looking for very distant supernovae and therefore high z values. A more accurate relation in this case is

$$d = \frac{cz}{H_0}\left(1 + \frac{1}{2}(1 - q_0)z\right)$$

Exam tip
You will not be examined on the formula
$$d = \frac{cz}{H_0}\left(1 + \frac{1}{2}(1 - q_0)z\right)$$
but knowing of its existence is crucial in understanding how the conclusion of an **accelerating universe** was reached.

We see that the deceleration parameter q_0 appears in this formula. So, *in theory*, the task looked easy: find distant objects, measure their distance d and red-shift z and use the data to determine q_0. The two groups chose to look at distant **Type Ia supernovae**. The nature of these will be discussed in more detail in Section **D4.4**.

Type Ia supernovae are very rare: only a few would be expected to occur in a galaxy every thousand years! The great discovery about Type Ia supernovae is that they all have the same peak luminosity and so may be used as standard candles. Figure **D.27** shows how the logarithm of the luminosity (in W) of a Type Ia supernova varies with time.

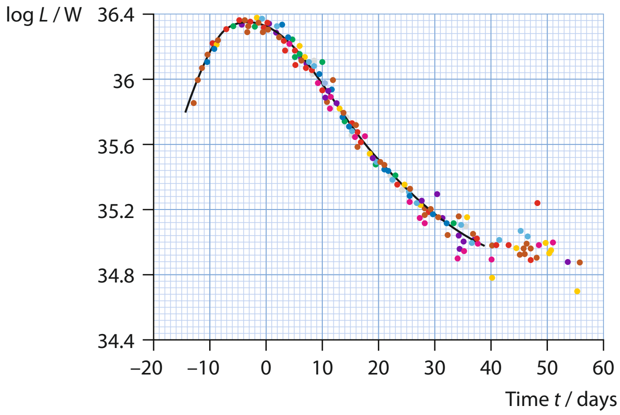


Figure D.27 Different Type Ia supernovae all have the same peak luminosity. (Adapted from graph 'Low redshift type 1a template lightcurve', Supernova Cosmology Project/Adam Reiss)

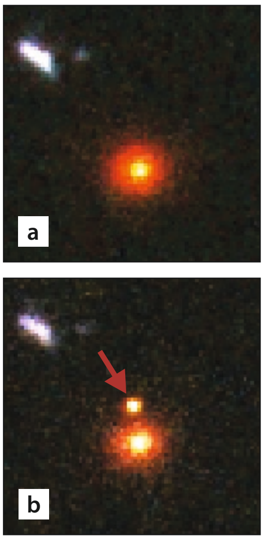


Figure D.28 Observation of a Type Ia supernova, **a** before and **b** after outburst.

Exam tip
A 'standard candle' refers to a star of known luminosity. Thus measuring its apparent brightness gives its distance.

The peak luminosity is a staggering 2×10^{36} W and falls off with time over a couple of months. If one is lucky enough to observe a Type Ia supernova from before the peak luminosity is reached, the peak apparent brightness b may be measured, and since the peak luminosity L is known we may find the distance to the supernova using $b = \frac{L}{4\pi d^2}$.

The task which looked easy in theory was formidable in practice, but a total of 45 supernovae were studied by the first group and 16 by the second (Figure **D.28**). Both groups were surprised to find that the deceleration parameter came out negative. This meant that the rate at which distant objects are moving away from us is **increasing**: the universe is **accelerating**. Perlmutter, Schmidt and Riess shared the 2011 Nobel Prize in Physics for this extraordinary discovery.

Worked example

D.18 Show that at a temperature $T = 10^{10}$ K there is enough thermal energy to create electron–positron pairs.

The thermal energy corresponding to $T = 10^{10}$ K is $E_k \approx \frac{3}{2}kT = 1.5 \times 1.38 \times 10^{-13}$ J $= 1.3$ MeV. The **rest energy** of an electron is $m_e c^2 \approx 0.5$ MeV, so the thermal energy $E_k \approx 1.3$ MeV is enough to produce a pair.

Nature of science

Occam's razor

Any theory of the origin of the universe must fit with the data available. The principle of **Occam's razor** says that the simplest explanation is likely to be the best. In the debate between conflicting cosmological theories, an expanding universe and the existence of a background radiation were predicted by the Big Bang theory. The red-shift in the light from distant galaxies was evidence for expansion, but it was the discovery of the cosmic microwave background radiation that led to the acceptance of the Big Bang theory. Although this theory explains many aspects of the universe as we see it now, it cannot explain what happened at the very instant the universe was created.

? Test yourself

43 **a** State and explain **Hubble's law**.
b Explain how this law is evidence for an expanding universe.

44 Some galaxies actually show a blue-shift, indicating that they are moving towards us. Discuss whether this violates Hubble's law.

45 Galaxies are affected by the gravitational pull of neighbouring galaxies and this gives rise to what are called **peculiar** velocities. Typically these are about $500\,\mathrm{km\,s^{-1}}$. Estimate how far away a galaxy should be so that its velocity of recession due to the expanding universe equals its peculiar velocity.

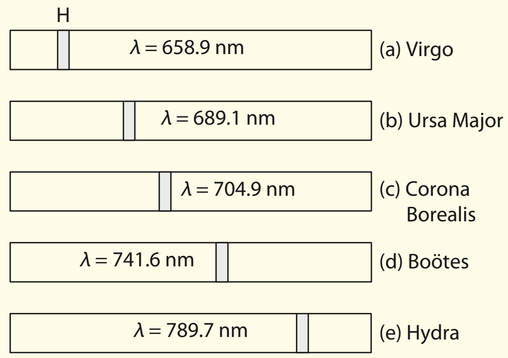
46 A student explains the expansion of the universe as follows: 'Distant galaxies are moving at high speeds into the vast expanse of empty space.' Suggest what is wrong with this statement.

47 It is said that the Big Bang started everywhere in space. Suggest what this means.

48 In the context of the Big Bang theory, explain why the question 'what existed before the Big Bang?' is meaningless.

49 Suppose that at some time in the future a detailed study of the Andromeda galaxy and all the nearby galaxies in our Local Group will be possible. Discuss whether this would help in determining Hubble's constant more accurately.

50 The diagram shows two lines due to calcium absorption in the spectra of five galaxies, ranging from the nearby Virgo to the very distant Hydra. Each diagram gives the wavelength of the H (hydrogen) line. The wavelength of the H line in the lab is 656.3 nm.
Using Hubble's law, find the distance to each galaxy. (Use $H = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.)



51 Take Hubble's constant H at the present time to be $68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.
a Estimate at what distance from the Earth the speed of a receding galaxy is equal to the speed of light.
b Suggest what happens to galaxies that are beyond this distance.
c The theory of special relativity states that nothing can exceed the speed of light. Suggest whether the galaxies in **b** violate relativity.

52 Discuss **three** pieces of evidence that support the Big Bang model of the universe.

53 A particular spectral line, when measured on Earth, corresponds to a wavelength of $4.5 \times 10^{-7}\,\mathrm{m}$. When received from a distant galaxy, the wavelength of the same line is measured to be $5.3 \times 10^{-7}\,\mathrm{m}$.
a Calculate the red-shift for this galaxy.
b Estimate the speed of this galaxy relative to the Earth.
c Estimate the distance of the galaxy from the Earth. (Take $H = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.)

54 Explain why the inverse of the Hubble constant, $\frac{1}{H}$, is taken to be an estimate of the 'age of the universe'. Estimate how old the universe would be if $H = 500\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (close to Hubble's original value).

55 Explain why the Hubble constant sets an **upper bound** on the age of the universe.

56 Explain why Hubble's law does not imply that the Earth is at the centre of the universe.

57 The temperature of the cosmic microwave background radiation as measured from the Earth is about 2.7 K.

a What is the significance of this radiation?

b What would be the temperature of the CMB radiation as measured by an observer in the Andromeda galaxy, 2.5 million light years away?

58 **a** Draw a sketch graph to show the variation of the CMB radiation intensity with wavelength.

b Calculate the peak wavelength corresponding to a CMB radiation temperature of 2.72 K.

59 Predict what will happen to the temperature of the CMB radiation if:

a the universe keeps expanding forever

b the universe starts to collapse.

60 **a** State what is meant by **red-shift**.

b Describe the mechanism by which the observed red-shift in light from distant galaxies is formed.

c Show that the distance d of a galaxy with a red-shift of z is given by $d = \frac{cz}{H_0}$.

d Calculate the distance of a galaxy whose red-shift is 0.18, using $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

e Estimate the size of the universe now, relative to its size when the light in **d** was emitted.

61 The wavelength of a particular spectral line measured in the laboratory is 486 nm. The same line observed in the spectrum of a distant galaxy is shifted by 15 nm.

a Estimate the distance of the galaxy, using $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

b Estimate the size of the universe now, relative to its size when the light in **a** was emitted.

62 A photon is emitted at a time when the size of the universe was 85% of its present size. Estimate the distance from the Earth of the point from which the photon was emitted, using $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

63 State the property of Type Ia supernovae that is significant in distance measurements in cosmology.

64 **a** State what is meant by the **accelerating universe**.

b Suggest why the universe was expected to decelerate rather than accelerate.

c Outline how Type Ia supernovae were used to discover the acceleration of the universe.

d Explain why it is important to observe such a supernova starting before it reaches its peak luminosity.

65 It is said that distant supernovae appear dimmer than they would in a decelerating universe. Explain this statement.

66 It was stated in the text that Type Ia supernovae are very rare (a few in a galaxy every thousand years). Suggest how two research groups were able to observe over 50 such supernovae in the space of just a few years.

Learning objectives

- Understand and apply the Jeans criterion.
- Describe nuclear fusion in stars.
- Describe nucleosynthesis off the main sequence.
- Distinguish and describe Type Ia and Type II supernovae.

D4 Stellar processes (HL)

This section deals with the birth, evolution and death of stars, and with their role in **nucleosynthesis,** the production of elements through fusion and neutron absorption. The section closes with a discussion of supernovae and the role of Type Ia supernovae as standard candles.

D4.1 The Jeans criterion

Interstellar space (the space between stars) consists of gas and dust at a density of about $10^{-21}\,\mathrm{kg\,m^{-3}}$. This amounts to about one atom of hydrogen in every cubic centimetre of space. The gas is mainly hydrogen (about 74% by mass) and helium (25%), with other elements making up the remaining 1%. Whenever the gravitational energy of a given mass of gas exceeds the average kinetic energy of the random thermal motion of its molecules, the gas becomes unstable and tends to collapse:

$$\frac{GM^2}{R} \geq \frac{3}{2}NkT$$

where k isBoltzmann's constant, T is temperature and N is the number of particles. R is the radius of the gas cloud and M its mass. This is known as the **Jeans criterion**.

Stars formed (and continue to be formed) when rather cool gas clouds in the interstellar medium ($T \approx 10$–$100\,\text{K}$) of sufficiently large mass (large enough to satisfy the Jeans criterion) collapsed under their own gravitation. In the process of contraction, the gas heated up. Typically, the collapsing gas would break up into smaller clouds, resulting in the creation of more than one star. When the temperature rises sufficiently for visible light to be emitted, a star so formed is called a **protostar** (see Figure **D.29**).

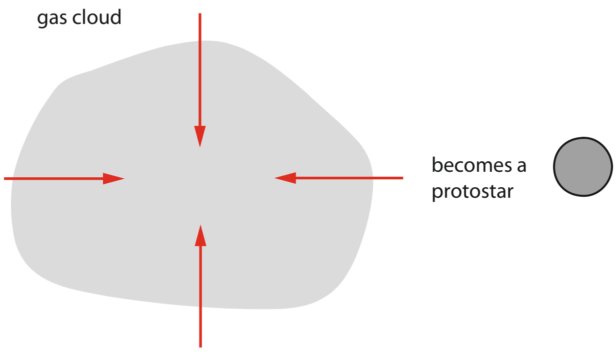


Figure D.29 The formation of a protostar out of a collapsing cloud of gas.

Worked examples

D.19 Show that the Jeans criterion can be rewritten as $M^2 = \frac{3}{4\pi\rho}\left(\frac{3kT}{2mG}\right)^3$, where ρ is the density of the gas and m is the mass of a particle of the gas. (The right hand side is known as the square of the Jeans mass.)

Cube each side of the Jeans criterion equation to find

$$\left(\frac{GM^2}{R}\right)^3 = \left(\frac{3kMT}{2m}\right)^3 \quad \Rightarrow \quad M^2 = \left(\frac{3}{4\pi\rho}\right)\left(\frac{3kT}{2mG}\right)^3$$

using the definition of density and $M = Nm$, where m is the mass of one molecule.

D.20 Take the density of interstellar gas in a cloud to be about 100 atoms of hydrogen per cm^3. Estimate the smallest mass this cloud can have for it to become unstable and begin to collapse when $T = 100\,\text{K}$.

The density is

$$\frac{100 \times 1.67 \times 10^{-27}\,\text{kg}}{10^{-6}\,\text{m}^3} = 1.67 \times 10^{-19}\,\text{kg}\,\text{m}^{-3}$$

With $T = 100\,\text{K}$ in the Jeans criterion we find (see Worked example **D.19**)

$$M \approx 3.0 \times 10^{33}\,\text{kg} = 1.5 \times 10^3\,M_\odot$$

Such a large gas cloud might well break up, forming more than one star.

D4.2 Nuclear fusion

We saw in Section **D1.2** that the main series of nuclear fusion reactions taking place in the cores of main-sequence stars is the proton–proton cycle:

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{1}e^{+} + \nu$$

$$^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

$$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + 2^{1}_{1}H$$

In this cycle, the net effect is to turn four hydrogen nuclei into one helium nucleus.

For stars more massive than our Sun, there is a second way to fuse hydrogen into helium. This is the so-called **CNO cycle**, described by the following series of fusion reactions:

$$^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N + \gamma$$

$$^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}e^{+} + \nu$$

$$^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N + \gamma$$

$$^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O + \gamma$$

$$^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{1}e^{+} + \nu$$

$$^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$$

Exam tip
The CNO cycle applies to more massive main-sequence stars and does not produce elements heavier than helium.

Notice that the net effect is to turn four hydrogen nuclei into one helium nucleus, just like the proton–proton cycle; the heavier elements produced in intermediate stages are all used up. The carbon nucleus has a charge of +6, so the barrier that must be overcome for carbon to fuse is much higher. This requires higher temperatures.

The proton–proton cycle and the CNO cycle are both main-sequence star processes. What happens beyond the main sequence?

The first element to be produced as a star leaves the main sequence and enters the red giant stage is carbon, through the **triple alpha process**:

Exam tip
The triple alpha process is how we end up with white dwarfs with a carbon core.

$$^{4}_{2}He + ^{4}_{2}He \rightarrow ^{8}_{4}Be + \gamma$$

$$^{4}_{2}He + ^{8}_{4}Be \rightarrow ^{12}_{6}C + \gamma$$

This happens to stars with masses up to eight solar masses. The star will shed most of its mass in a planetary nebula and end up as a white dwarf with a core of carbon.

For stars even more massive than this, helium fuses with carbon to produce oxygen:

$$^{4}_{2}He + ^{12}_{6}C \rightarrow ^{16}_{8}O + \gamma$$

In even more massive stars, neon, sodium and magnesium are produced:

$$^{12}_{6}C + ^{12}_{6}C \rightarrow ^{20}_{10}Ne + ^{4}_{2}He$$

$$^{12}_{6}C + ^{12}_{6}C \rightarrow ^{23}_{11}Na + ^{1}_{1}H$$

$$^{12}_{6}C + ^{12}_{6}C \rightarrow ^{23}_{12}Mg + ^{1}_{0}n$$

Silicon is then produced by the fusion of oxygen:

$$^{16}_{8}O + ^{16}_{8}O \rightarrow ^{28}_{14}Si + ^{4}_{2}He$$

The process continues until iron is formed. This creates an onion-like layered structure in the star, with progressively heavier elements as we move in towards the centre. Fusion cannot produce elements heavier than iron, since the binding energy per nucleon peaks near iron and further fusion is not energetically possible. Thus, a massive star ends its cycle of nuclear reactions with iron at its core, surrounded by progressively lighter elements, as shown in Figure **D.30**.

Exam tip
You should be able to explain why fusion ends with the production of iron.

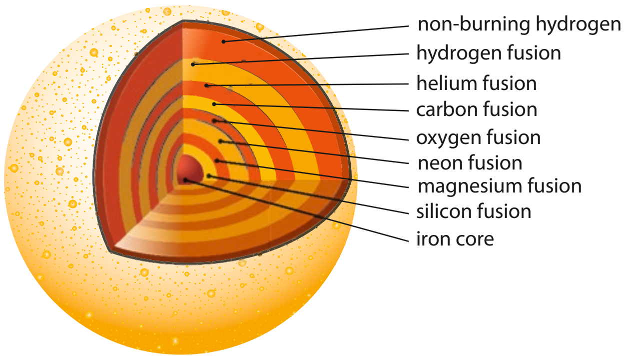


Figure D.30 The central core of a fully evolved massive star consists of iron with layers of lighter elements surrounding it.

Worked example

D.21 Our Sun emits energy at a rate (luminosity) of about 3.9×10^{26} W. Estimate the mass of hydrogen that undergoes fusion in one year. Assuming that the energy loss is maintained at this rate, find the time required for the Sun to convert 12% of its hydrogen into helium. (Mass of Sun $= 1.99 \times 10^{30}$ kg.)

Assuming the proton–proton cycle as the reaction releasing energy (by hydrogen fusion), the energy released per reaction is about 3.98×10^{-12} J. Since the luminosity of the Sun is 3.9×10^{26} W, it follows that the number of fusion reactions required per second is

$$\frac{3.9 \times 10^{26}}{3.98 \times 10^{-12}} = 9.8 \times 10^{37}$$

For every such reaction, four hydrogen nuclei fuse to helium and thus the mass of consumed hydrogen is $9.8 \times 10^{37} \times 4 \times 1.67 \times 10^{-27}\,\text{kg}\,\text{s}^{-1} = 6.5 \times 10^{11}\,\text{kg}\,\text{s}^{-1} = 2 \times 10^{19}$ kg per year. At the time of its creation, the Sun consisted of 75% hydrogen, corresponding to a mass of $0.75 \times 1.99 \times 10^{30}\,\text{kg} = 1.5 \times 10^{30}$ kg. The limit of 12% results in a hydrogen mass to be fused of 1.8×10^{29} kg. The time for this mass to fuse is thus

$$\frac{1.8 \times 10^{29}}{6.5 \times 10^{11}}\,\text{s} = 2.8 \times 10^{17}\,\text{s}$$
$$= 8.9 \times 10^{9}\,\text{yr}$$

Since the Sun has existed for about 5 billion years, it still has about 4 billion years left in its life as a main-sequence star.

One application of the mass–luminosity relation is to estimate the lifetime of a star on the main sequence. Since the luminosity is the power radiated by the star, we may write that

$$\frac{E}{T} \propto M^{3.5}$$

where E is the total energy radiated by the star and T is the time in which this happens. For the purposes of an estimate, we may assume that the total energy that the star can radiate comes from converting *all* its mass into energy according to Einstein's formula, $E = Mc^2$. Thus

$$\frac{E}{T} \propto M^{3.5} \quad \Rightarrow \quad \frac{Mc^2}{T} \propto M^{3.5} \Rightarrow T \propto M^{-2.5}$$

This means that the lifetimes of two stars are approximately related by

$$\frac{T_1}{T_2} = \left(\frac{M_2}{M_1}\right)^{2.5}$$

Worked example

D.22 Our Sun will spend about 10^{10} yr on the main sequence. Estimate the time spent on the main sequence by a star whose mass is 10 times the mass of the Sun.

We know that $\frac{T_1}{T_2} = \left(\frac{M_2}{M_1}\right)^{2.5}$. Hence, $\frac{T}{T_\odot} = \left(\frac{M_\odot}{10M_\odot}\right)^{2.5} \quad \Rightarrow \quad T = \frac{10^{10}}{10^{2.5}} \approx 3 \times 10^7$ yr.

D4.3 Nucleosynthesis of the heavy elements

All of the hydrogen and most of the helium in the universe were produced at the very earliest moments in the life of the universe. Everything else was made in stars in the course of stellar evolution. In Section **D4.2** we learned that nuclear fusion reactions in stellar cores produce the elements up to iron. So how are the rest of the elements in the periodic table produced?

The answer lies in neutron absorption by nuclei – **neutron capture**. When a nucleus absorbs a neutron it becomes an isotope of the original nucleus. This isotope is usually unstable and will decay. The issue is whether there is enough time for this decay to occur before the isotope absorbs yet another neutron. In what is referred to as an **s-process** (s for 'slow'), the isotope does have time to decay because the number of neutrons present is small. This happens in stars where relatively small numbers of neutrons are produced in the fusion reactions discussed in Section **D4.2**. The isotope will undergo a series of decays, including beta decay, in which the atomic number is increased by one, thus producing a new element. This process accounts for the production of about half of the nuclei above iron, and ends with the production of bismuth 209.

By contrast, in the presence of very large numbers of neutrons, nuclei that absorb neutrons do not have time to decay. In an **r-process** (r for 'rapid'), they keep absorbing neutrons one by one, forming very heavy, neutron-rich isotopes. This cannot happen inside a star but it does happen during a supernova explosion. These neutron-rich isotopes are then hurled into space by the supernova, where they can undergo beta decay, producing nuclei of higher atomic number.

Beta decay is not the only way to turn a neutron into a proton and hence increase the atomic number. In supernova explosions, massive numbers of neutrinos are produced. A neutron may absorb a neutrino and turn into a proton according to the reaction

$${}^{1}_{0}n + \nu \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e$$

D4.4 Type Ia and Type II supernovae

The supernovae we learned about in Section **D2.8** referred to the explosion of red supergiant stars. These are called **Type II supernovae**. Another type of supernovae, called **Type Ia supernovae**, involve a different mechanism.

Consider a **binary star** system in which one of the stars is a white dwarf. This star may attract material from its companion star. Mass falling into the white dwarf may increase the white dwarf's mass beyond the Chandrasekhar limit, so nuclear fusion reactions may start again in its core. The resulting sudden release of energy appears as a sudden increase in the luminosity of the white dwarf – that is, a supernova. Unlike Type II supernovae, Type Ia supernovae show no hydrogen lines.

Figure **D.31** shows the remnants of two supernovae.

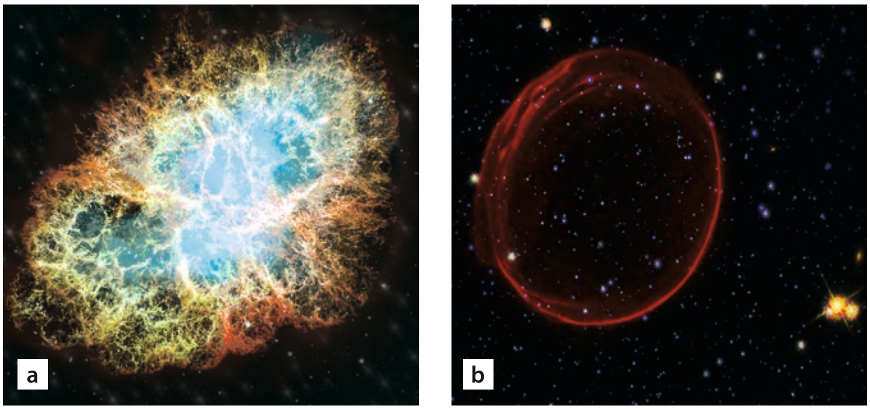


Figure D.31 **a** The Crab nebula, the remnant of a Type II supernova observed by the Chinese in 1054 and possibly by Native Americans. It was visible for weeks, even during daytime. **b** This majestically serene bubble of gas in space is the remnant of the Type Ia supernova SNR 0509, which exploded about 400 years ago.

As noted in Section **D3.5**, an important property of Type Ia supernovae is that they all have the same peak luminosity and so may be used as 'standard candles'. By measuring their apparent brightness at the peak, we may calculate their distance from $d = \sqrt{\frac{L}{4\pi b}}$ (see Section **D1.5**).

Apart from the mechanism producing them, the two types of supernovae also differ in that Type Ia supernovae do not have hydrogen absorption lines in their spectra, whereas Type II do. They also differ in the way the luminosity falls off with time (Figure **D.32**).

Exam tip

Differences in the two types of supernovae:

Type Ia

- do not have hydrogen lines in their spectra
- are produced when mass from a companion star accretes onto a white dwarf, forcing it to exceed the Chandrasekhar limit
- have a luminosity which falls off sharply after the explosion.

Type II

- have hydrogen lines in their spectra
- are produced when a massive red supergiant star explodes
- have a luminosity which falls off gently after the explosion.

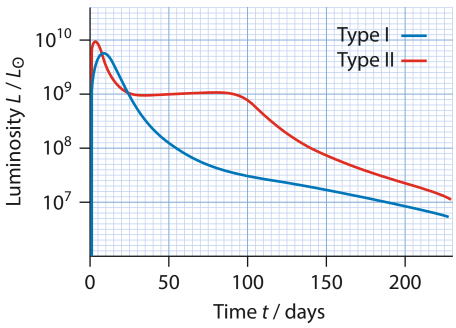


Figure D.32 Type Ia and Type II supernovae show different graphs of luminosity versus time.

Nature of science

Observation and deduction

Stellar spectra show us the elements present in stars' outer layers, but how do we explain how these elements came to be formed? Astrophysicists have shown how nuclear fusion reactions in the stars produce energy and also build up the elements. Modelling and computer simulations have resulted in a picture that agrees very well with observation. The time scales of stellar processes are much too long for us to directly observe the phenomena of stellar evolution, but the presence of stars of different ages and at different stages of evolution allows a comparison of observations and theoretical predictions.

? Test yourself

67 Describe the **Jeans criterion**.

68 Describe how a gas and dust cloud becomes a protostar.

69 Explain whether star formation is more likely to take place in cold or hot regions of interstellar space.

70 Explain why a star at the top left of the main sequence will spend much less time on the main sequence than a star at the lower right.

71 Show that a star that is twice as massive as the Sun has a lifetime that is 5.7 times shorter than that of the Sun.

72 Using the known luminosity of the Sun and assuming that it stays constant during the Sun's lifetime, which is estimated to be 10^{10} yr, calculate the mass this energy corresponds to according to Einstein's mass–energy formula.

73 Suggest why the depletion of hydrogen in a star is such a significant event.

74 Evolved stars that have left the main sequence have an onion-like layered structure. Outline how this structure is created.

75 Describe the nuclear reactions taking place in a star of one solar mass:
- **a** while the star is on the main sequence
- **b** after it has left the main sequence.

76 Describe the nuclear reactions taking place in a star of 20 solar masses:
- **a** while on the main sequence
- **b** after it has left the main sequence.

77 Explain why no elements heavier than iron are produced in stellar cores.

78 State the element that is the end product of:
- **a** the proton–proton cycle
- **b** the CNO cycle
- **c** the triple alpha process.

79 Distinguish between an **s-process** and an **r-process**.

80 Suggest why the production of heavier elements inside stars requires higher temperatures.

81 Describe how a Type Ia supernova is formed.

82 Describe how a Type II supernova is formed.

83 State **three** differences between Type Ia and Type II supernovae.

84 Suggest why hydrogen lines are expected in the spectra of Type II supernovae.

85 Compare and contrast the proton–proton and CNO cycles.

D5 Further cosmology (HL)

This section deals with some open questions in cosmology, questions that are the subject of intensive current research. These include the evidence for and the nature of dark matter and dark energy, fluctuations in the CMB, and the **rotation curves** of galaxies

Learning objectives

- Describe the cosmological principle.
- Understand the fluctuations in the CMB.
- Understand the cosmological origin of red-shift.
- Derive the critical density and understand its significance.
- Describe dark matter.
- Derive rotation curves and understand how they provide evidence for dark matter.
- Understand dark energy.
- Sketch the variation of the scale factor with time for various models.

D5.1 The cosmological principle

The universe appears to be full of structure. There are planets and moons in our solar system, there are stars in our galaxy, our galaxy is part of a **cluster of galaxies** and our cluster is part of an even bigger **supercluster** of galaxies.

If we look at the universe on a very large scale, however, we no longer see any structure. If we imagine cutting up the universe into cubes some 300 Mpc on a side, the interior of any one of these cubes would look much the same as the interior of any other, anywhere else in the universe. This is an expression of the so-called **homogeneity principle** in cosmology: on a large enough scale, the universe looks uniform.

Similarly, if we look in different directions, we see essentially the same thing. If we look far enough in any direction, we will count the same number of galaxies. No one direction is special in comparison with another. This leads to a second principle of cosmology, the **isotropy principle**. A related observation is the high degree of isotropy of the CMB.

> These two principles, homogeneity and isotropy, make up what is called the **cosmological principle**, which has had a profound role in the development of models of cosmology.

The cosmological principle implies that the universe has no edge (for if it did, the part of the universe near the edge would look different from a part far from the edge, violating the homogeneity principle). Similarly, it implies that the universe has no centre (for if it did, an observation from the centre would show a different picture from an observation from any other point, violating the principle of isotropy).

D5.2 Fluctuations in the CMB

We have noted several times that the CMB is uniform and isotropic. However, it is not perfectly so. There are small variations ΔT in temperature, of the order of $\frac{\Delta T}{T} \approx 10^{-5}$, where $T = 2.723\,\text{K}$ is the average temperature. These variations in temperature are related to variations in the density of the universe. In turn, variations in density are the key to how structures formed in the universe. With perfectly uniform temperature and density in the universe, stars and galaxies would not form.

> The anisotropies in the CMB are crucial in understanding the formation of structures.

In addition to helping us understand structures, CMB anisotropies are related to the geometry of the universe. There would be different degrees of anisotropy depending on whether the universe has positive, zero or negative curvature (see the section on the dependence of the **scale factor** on time later in this section). A number of investigations of CMB anisotropies have been carried out, using COBE, WMAP, the Planck satellite observatory and the Boomerang (Balloon Observations

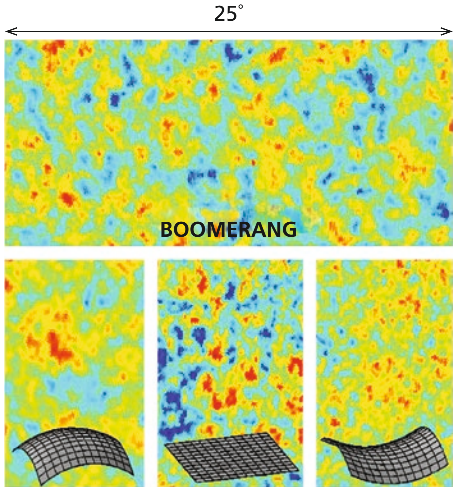


Figure D.33 Fluctuations in temperature are shown as differences in colour in this image from the Boomerang collaboration. Theoretical models using space of different curvatures are also shown. There is a clear match with the flat-universe case. © The Boomerang Collaboration.

Of Millimetric Extragalactic Radiation) collaboration. Figure **D.33** shows fluctuations in the CMB temperature obtained by the Boomerang collaboration, and three theoretical predictions of what that anisotropy should look like in models with positive, negative and zero **curvature** of space. Different colours correspond to different temperatures. Even judged by eye, the data appear to be consistent with the flat case.

Figure **D.34** is a spectacular map from the Planck satellite observatory, showing **CMB fluctuations** in temperature as small as a few millionths of a degree. This is a map of the radiation filling the universe when it was only about 380 000 years old.

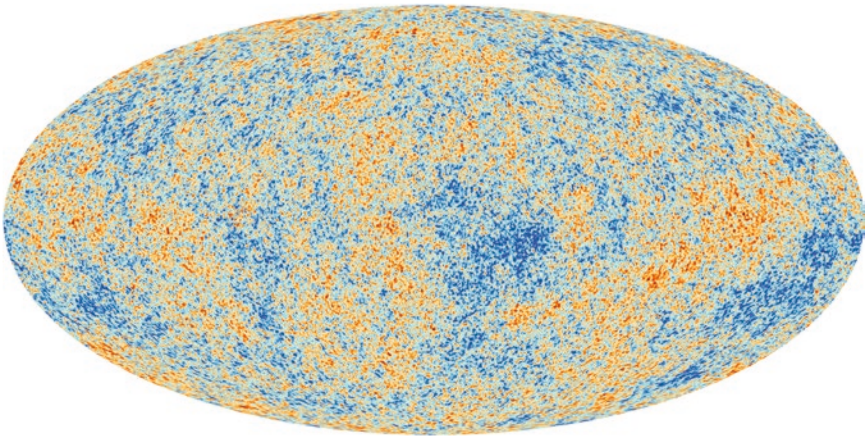


Figure D.34 Fluctuations in temperature of the CMB according to ESA's Planck satellite observatory. (©ESA and the Planck Collaboration, reproduced with permission)

Studies of CMB anisotropy also give crucial information on cosmological parameters such as the density of matter and energy in the universe.

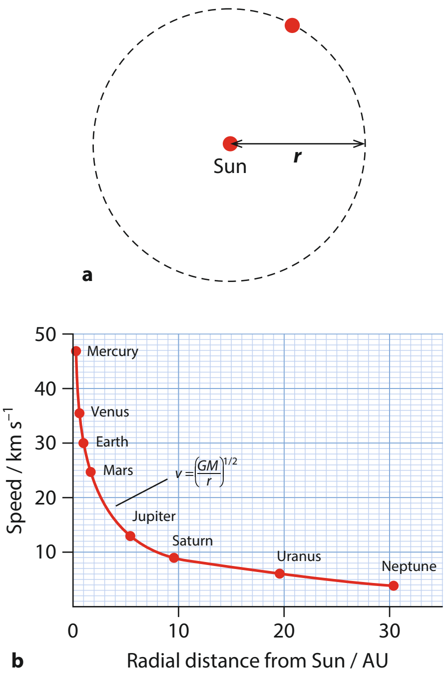


Figure D.35 **a** A particle orbiting a central mass. **b** The rotation curve of the particle in **a** shows a characteristic drop. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission)

D5.3 Rotation curves and the mass of galaxies

Consider a planet as it revolves about the Sun (Figure **D.35a**). In Topics **6** and **10** we determined, using $\frac{GMm}{r^2} = \frac{mv^2}{r}$, that the speed of a planet a distance r from the Sun is $v = \sqrt{\frac{GM}{r}}$. This means that $v \propto \frac{1}{\sqrt{r}}$. Plotting rotational speed against distance gives what is called a **rotation curve**, as shown in Figure **D.35b**.

Now consider a spherical mass cloud of uniform density (Figure **D.36a**). What is the speed of a particle rotating about the centre at a distance r? We can still use $v = \sqrt{\frac{GM}{r}}$, but now M stands for the mass in the spherical body up to a distance r from the centre. Since the density is constant we have that

$$\rho = \frac{M}{V} = \frac{M}{\frac{4\pi r^3}{3}} = \frac{3M}{4\pi r^3}$$

and hence

$$M = \frac{4\pi r^3 \rho}{3}$$

Thus

$$v = \sqrt{\frac{G4\pi r^3 \rho}{3r}}$$

and so $v \propto r$. The rotation curve is a straight line through the origin. This is valid for r up to R, the radius of the spherical mass cloud. Beyond R the curve is like that of Figure **D.35**.

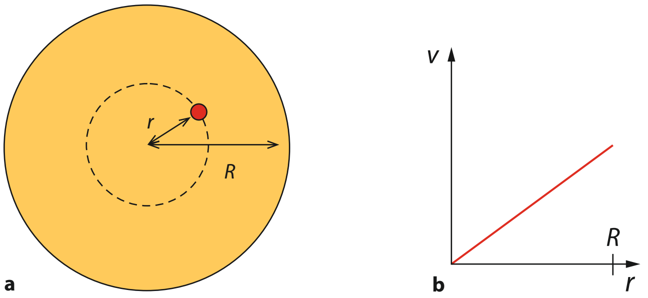


Figure D.36 **a** A particle orbiting around the centre of a uniform spherical cloud. **b** The rotation curve of the particle in **a**.

Worked example

D.23 Consider a system in which the mass varies with distance from the axis according to $M = kr$, where k is a constant. Derive the rotation curve for such a system.

We start with $v = \sqrt{\frac{GM}{r}}$. We get $v = \sqrt{\frac{Gkr}{r}} = \sqrt{Gk} =$ constant. The rotation curve would thus be a horizontal straight line.

Figure **D.37** shows the rotation curve of the Milky Way galaxy.

This rotation curve is not one that belongs to a large central mass, as in Figure **D.35**. Its main feature is the flatness of the curve at large distances from the centre. Notice that the flatness starts at distances of about 13 kpc. The central galactic disc has a radius of about 15 kpc. This means that there is substantial mass outside the central galactic disc. Furthermore, according to Worked example **D.23**, a flat curve corresponds to increasing mass with distance from the centre.

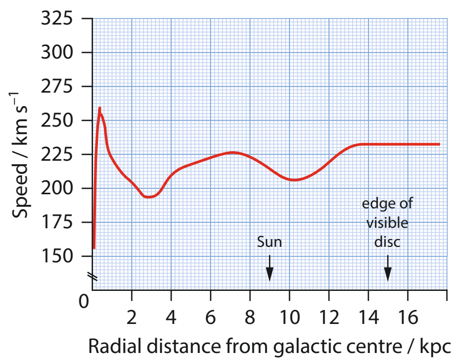


Figure D.37 The rotation curve of our galaxy shows a flat region, indicating the presence of matter far from the galactic disc. (Adapted from Combes, F. (1991) Distribution of CO in the Milky Way, Annual Review of Astronomy and Astrophysics, 29, pp.195–237)

> The flat part of the galaxy's rotation curve indicates substantial mass far from the centre.

If the mass were contained within a given distance, then past that distance the rotation curve should have dropped, as it does in Figure **D.35**. The problem is that no such drop is seen – but at the same time no such mass is visible. Arguments like this have led to the conclusion that there must be considerable mass at large distances past the galactic disc. This is **dark matter**: matter that is too cold to radiate and so cannot be seen. It is estimated that, in our galaxy, dark matter forms a spherical halo around the galaxy and has a mass that is about 10 times larger than the mass of all the stars in the galaxy!

D5.4 Dark matter

It is estimated that 85% of the matter in the universe is dark matter. It cannot be seen; we know of its existence mainly from its gravitational effects on nearby bodies.

What could dark matter be? It could be ordinary, cold matter that does not radiate – like, for example, brown dwarfs, black dwarfs or small planets. Collectively these are called MACHOs (MAssive Compact Halo Objects). This is matter consisting mainly of protons and neutrons, so it also called **baryonic matter**. The problem is that there are limits to how much baryonic matter there can be. The limit is at most 15%, so dark matter must also contain other, more exotic forms.

The class of non-baryonic objects which are candidates for dark matter are called WIMPs (for Weakly Interacting Massive Particles). Neutrinos fall into this class since they are known to have a small mass, although their tiny mass is not enough to account for all non-baryonic dark matter. Unconfirmed theories of elementary particle physics based mainly on the idea of **supersymmetry** (a proposed symmetry between particles with integral spin and particles with half-integral spin) predict the existence of various particles that would be WIMP candidates – but no such particles have been discovered.

So the answer to the question 'what is dark matter?' is mainly unknown at the moment.

D5.5 The cosmological origin of red-shift

In Section **D3.2** we derived the formula

$$\frac{\lambda}{\lambda_0} = \frac{R}{R_0}$$

where R_0 is the value of the scale factor at the time of emission of a photon of wavelength λ_0, R is the value of the scale factor at the present time (when the photon is received) and λ is the wavelength of the photon as measured at the present time. This gives a cosmological interpretation of the red-shift, rather than one based on the Doppler effect: the space in between us and the galaxies is stretching, so wavelengths stretch as well.

A direct consequence of this is on the temperature of the CMB radiation that fills the universe. The wavelength λ_0 corresponds to a CMB temperature of T_0. By the Wien displacement law,

$$\lambda_0 T_0 = \lambda T = \text{constant}$$

Therefore

$$\frac{\lambda}{\lambda_0} = \frac{T_0}{T}$$

This implies that

$$\frac{T_0}{T} = \frac{R}{R_0} \text{ or } T \propto \frac{1}{R}$$

This shows that, as the universe expands (that is, as R gets bigger), the temperature drops. This is why the universe is cooling down, and why the present temperature of the CMB is so low (2.7 K).

Worked example

D.24 The photons of CMB radiation observed today are thought to have been emitted at a time when the temperature of the universe was about 3.0×10^3 K. Estimate the size of the universe then compared with its size now.

From $T \propto \frac{1}{R}$ we find $\frac{T_0}{T} = \frac{R}{R_0}$, so $\frac{R}{R_0} = \frac{3.0 \times 10^3}{2.7} \approx 1100$, so the universe then was about 1100 times smaller.

One of the great problems in cosmology is to determine how the scale factor depends on time. We will look at this problem in the next two sections.

D5.6 Critical density

We begin the discussion in this section by solving a problem in Newtonian gravitation. Consider a spherical cloud of dust of radius r and mass M, and a mass m at the surface of this cloud which is moving away from the centre with a velocity v satisfying Hubble's law, $v = H_0 r$; see Figure **D.38**.

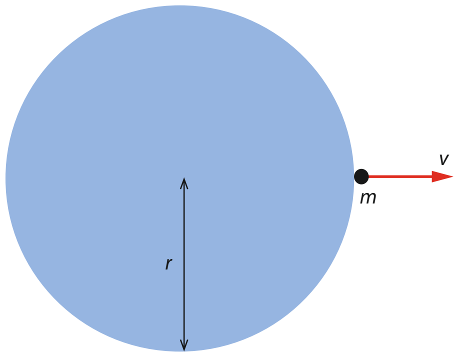


Figure D.38 Estimating critical density.

The total energy of the mass m is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

If ρ is the density of the cloud, then $M = \rho\frac{4}{3}\pi r^3$. Using this together with $v = H_0 r$, we find

$$E = \frac{1}{2}mr^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$$

The mass m will continue to move away if its total energy is positive. If the total energy is zero, the expansion will halt at infinity; if it is negative, contraction will follow the expansion. The sign of the term $\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$, that is, the value of ρ relative to the quantity

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 10^{-26}\,\text{kg m}^{-3}$$

determines the long-term behaviour of the cloud.

This quantity is known as the **critical density**. We have only derived it in a simple setting based on Newton's gravitation. What does it have to do with the universe? We discuss this in the next section.

Exam tip
You must be able to derive the formula for the critical density.

D5.7 The variation of the scale factor with time

In 1915 Einstein published his general theory of relativity, replacing Newton's theory of gravitation with a revolutionary new theory in which the rules of geometry are dictated by the distribution of mass and energy in the universe. Applying Einstein's theory to the universe as a whole results in equations for the scale factor R, and solving these gives the dependence of R on time. Einstein himself believed in a static universe – that is, a universe with R = constant. His equations, however, did not give a constant R. So he modified them, adding his famous **cosmological constant** term, Λ, to make R constant (see Figure **D.39**).

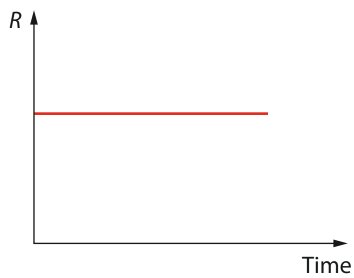


Figure D.39 A model with a constant scale factor. Einstein introduced the cosmological constant Λ in order to make the universe static.

In this model there is no Big Bang, and the universe always has the same size. This was before Hubble discovered the expanding universe. Einstein missed the great chance of theoretically predicting an expanding universe before Hubble; he later called adding the cosmological constant ‘the greatest blunder of [his] life’. This constant may be thought of as related to a ‘vacuum energy’, energy that is present in all space. The idea fell into obscurity for many decades but it did not go away: it was to make a comeback with a vengeance much later! It is now referred to as **dark energy**.

The first serious attempt to determine how R depends on time was made by the Russian mathematician Alexander Friedmann (1888–1925). Friedmann applied the Einstein equations and realised that there were a number of possibilities: the solutions depend on how much matter and energy the universe contains.

We define the **density parameters** Ω_m and Ω_Λ, for matter and dark energy respectively, as

$$\Omega_m = \frac{\rho_m}{\rho_c} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}$$

where ρ_m is the actual density of matter in the universe, ρ_c is the critical density derived in Section **D5.6** and ρ_Λ is the density of dark energy. The Friedmann equations give various solutions depending on the values of Ω_m and Ω_Λ. Deciding which solution to pick depends crucially on these values, which is why cosmologists have expended enormous amounts of energy and time trying to accurately measure them.

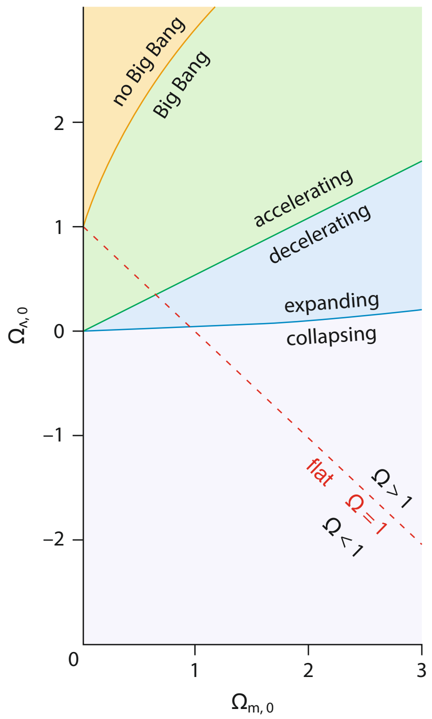


Figure D.40 There are various possibilities for the evolution of the universe depending on how much energy and mass it contains. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission.)

Figure **D.40** is a schematic representation of the possibilities; the subscript 0 indicates the values of these parameters at the present time. There are four regions in the diagram. The shape of the graph of scale factor versus time is different from region to region.

Notice the red dashed line: for models above the line, the geometry of the universe resembles that of the surface of a sphere. Those below the line have a geometry like that of the surface of a saddle. Points on the line imply a flat universe in which the rules of Euclidean geometry apply. Figure **D.41** illustrates these three models.

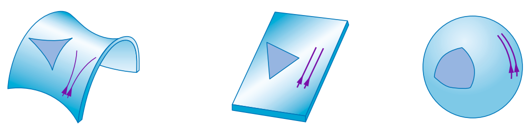


Figure D.41 Three models with different curvatures. **a** In negative-curvature models, the angles of a triangle add up to less than 180° and initially parallel lines eventually diverge. **b** Ordinary flat (Euclidean) geometry. **c** Positive curvature, in which the angles of a triangle add up to more than 180° and initially parallel lines eventually intersect. (From M. Jones and R. Lambourne, An Introduction to Galaxies and Cosmology, Cambridge University Press, in association with The Open University, 2004. © The Open University, used with permission)

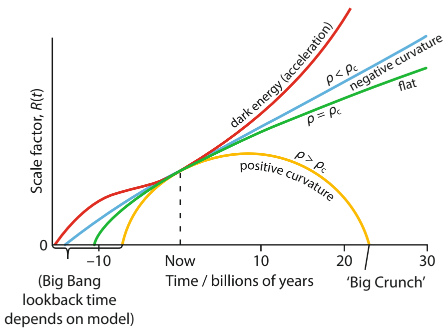


Figure D.42 Solutions of Einstein’s equations for the evolution of the scale factor. The present time is indicated by ‘now’. Notice that the estimated age of the universe depends on which solution is chosen. Three models assume zero dark energy; the one shown by the red line does not.

Of the very many possibilities, we will be interested in just four cases. The first three correspond to $\Omega_\Lambda = 0$ (they are of mainly historical interest, because observations favour $\Omega_\Lambda \neq 0$). These are shown as the orange, green and blue lines in Figure **D.42**.

In all three cases the scale factor starts from zero, implying a Big Bang. In one possibility (orange line), $R(t)$ starts from zero, increases to a maximum value and then returns to zero; that is, the universe collapses after an initial period of expansion. This is called the **closed model**, and corresponds to $\Omega_m > 1$, i.e. $\rho_m > \rho_c$. The second possibility corresponds to $\Omega_m < 1$, i.e. $\rho_m < \rho_c$. Here the scale factor $R(t)$ increases without limit – the universe continues to expand forever. This is called the **open model**. The third possibility is that the universe expands forever, but with a decreasing rate of expansion, becoming zero at infinite time. This is called the **critical model** and corresponds to $\Omega_m = 1$. The density of the universe in this case is equal to the critical density: $\rho_m = \rho_c$.

Keep in mind that these three models have $\Omega_\Lambda = 0$ and so are *not* consistent with observations. The fourth case, the red line in Figure **D.42**, is the one that agrees with current observations. Data from the Planck satellite observatory (building on previous work by WMAP, Boomerang and COBE) indicate that $\Omega_m \approx 0.32$ and $\Omega_\Lambda \approx 0.68$. This implies that $\Omega_m + \Omega_\Lambda \approx 1$, and corresponds to the red dashed line in Figure **D.42**. This is consistent with the analysis of the Boomerang data that we discussed in Section **D5.2**, and means that at present our universe has a flat geometry and is expanding forever at an accelerating rate, and that 32% of its mass–energy content is matter and 68% is dark energy.

D5.8 Dark energy

In Section **D3.5** we saw how analysis of distant Type Ia supernovae led to the conclusion that the expansion of the universe is accelerating. This ran contrary to expectations: gravity should be slowing the distant galaxies down. We also saw that an accelerating universe demands a non-zero value of the cosmological constant, which in turn implies the presence of a 'vacuum energy' that fills all space; this energy has been called dark energy.

The presence of this energy creates a kind of repulsive force that not only counteracts the effects of gravity on a large scale but actually dominates over it, causing acceleration in distant objects rather than the expected deceleration. The domination of the effects of dark energy over gravity appears to have started about 5 billion years ago.

There is now convincing evidence that $\Omega_m + \Omega_\Lambda \approx 1$, based on detailed studies of anisotropies in CMB radiation undertaken by COBE, WMAP, the Boomerang collaboration and the Planck satellite observatory. Data from Planck indicate that the mass–energy density of the universe consists of approximately 68% dark energy, 27% dark matter and 5% ordinary matter. This means that we understand just 5% of the mass–energy of the universe! These facts – along with the discovery (announced in March 2014 but still not confirmed) of gravitational waves, supporting another important part of the Big Bang model (inflation) – make these very exciting times for cosmology!

Nature of science

Cognitive bias

When interpreting experimental results, it is tempting to dismiss or find ways to explain away results that do not fit with the hypotheses. In the late 20th century, most scientists believed that the expansion of the universe must be slowing down because of the pull of gravity.

Evidence from the analysis of Type Ia supernovae in 1998 showed that the expansion of the universe is accelerating – a very unexpected result. Corroboration from other sources has led to the acceptance of this result, with the proposed dark energy as the cause of the acceleration.

? Test yourself

86 **a** Outline what is meant by the **cosmological principle**.

b Explain how this principle may be used to deduce that the universe:

i has no centre

ii has no edge.

87 State **two** pieces of observational evidence that support the cosmological principle.

88 **a** Outline what is meant by the **scale factor of the universe**.

b Sketch a graph to show how the scale factor of the universe varies with time for a model with zero cosmological constant and a density greater than the critical density.

c Draw another graph to show the variation of the CMB temperature for the model in **b**.

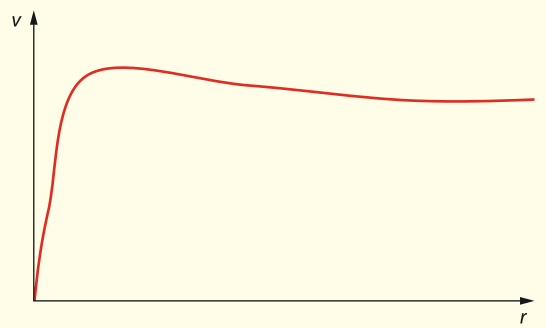
89 Sketch and label **three** graphs to show how the scale factor of the universe varies with time for models with zero cosmological constant. Use your graphs to explain why the three models imply different ages of the universe.

90 **a** Derive the rotation curve formula (showing the variation of speed with distance) for a mass distribution with a uniform density.

b Draw the rotation curve for **a**.

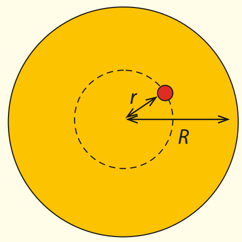
91 **a** Derive the rotation curve formula for a spherical distribution of mass in a galaxy that varies with the distance r from the centre according to $M(r) = kr$, where k is a constant.

b By comparing your answer with the rotation curve below, suggest why your rotation curve formula implies the existence of matter away from the centre of the galaxy.

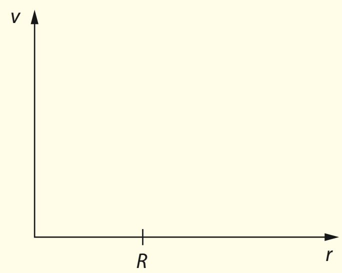


92 Sketch a graph to show the variation of the scale factor for a universe with a non-zero cosmological constant.

93 The diagram below shows a spherical cloud of radius R whose mass is distributed with constant density.



a A particle of mass m is at distance r from the centre of the cloud. On a copy of the axes below, draw a graph to show the expected variation with r of the orbital speed v of the particle for $r < R$ and for $r > R$.



b Describe one way in which the rotation curve of our galaxy differs from your graph.

94 **a** What do you understand by the term **dark matter**?

b Give three possible examples of dark matter.

95 Distinguish between **dark matter** and **dark energy**.

96 Explain why, in an accelerating universe, distant supernovae appear dimmer than expected.

97 **a** Outline what is meant by the **anisotropy of the CMB**.

b State what can be learned from studies of CMB anisotropies.

98 State how studies of CMB anisotropy lead to the conclusion that the universe is flat.

99 Use Figure **D.40** to suggest whether current data support a model with a negative cosmological constant.

100 Derive the dependence $T \propto \frac{1}{R}$ of the temperature T of the CMB on the scale factor R of the universe.

101 **a** Outline what is meant by the critical density.

b Show using Newtonian gravitation that the critical density of a cloud of dust expanding according to Hubble's law is given by $\rho_c = \frac{3H^2}{8\pi G}$.

c Current data suggest that $\Omega_m = 0.32$ and $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. alculate the matter density of the universe.

102 **a** Suggest why determination of the mass density of the universe is very difficult.

b Estimate how many hydrogen atoms per m^3 cubic metre a critical density of $\rho_c \approx 10^{-26}\,\mathrm{kg\,m^{-3}}$ represents.

103 **a** State what is meant by an accelerating of the universe.

b Draw the variation of the scale factor with time for an accelerating model.

c Use your answer to draw the variation of temperature with time in an accelerating model.

104 Explain, with the use of two-dimensional examples if necessary, the terms **open** and **closed** as they refer to cosmological models. Give an example of a space that is finite without a boundary and another that is finite with a boundary.

105 The density parameter for dark energy is given by $\rho_\Lambda = \frac{\Lambda c^2}{3H_0^2}$. Deduce the value of the cosmological constant, given $\Omega_\Lambda = 0.68$ and $H_0 = 68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

106 **a** List three reasons why Einstein's prediction of a constant scale factor is not correct.

b Identify a point on the diagram in Figure **D.40** where Einstein's model is located.

107 The Friedmann equation states that

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda c^2}{8\pi G}\right) - \frac{kc^2}{R^2}$$

where the parameter k determines the curvature of the universe: $k > 0$ implies a closed universe, $k < 0$ an open universe and $k = 0$ a flat universe. Deduce the geometry of the universe given that $\Omega_m + \Omega_\Lambda = 1$.

Exam-style questions

1 **a** Describe what is meant by:

i luminosity **[1]**

ii apparent brightness. **[1]**

Achernar is a main-sequence star with a mass equal to 6.7 solar masses. Its apparent brightness is $1.7 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ and its surface temperature is 2.6 times the Sun's temperature. The luminosity of the Sun is $3.9 \times 10^{26}\,\mathrm{W}$.

b State **one** characteristic of main-sequence stars. **[1]**

c Estimate for Achernar:

i its luminosity **[2]**

ii its parallax angle **[2]**

iii its radius in terms of the solar radius. **[3]**

d **i** Describe the **method of parallax** for measuring distances to stars. **[4]**

ii Suggest whether the parallax method can be used for Achernar. **[1]**

2 a Suggest how the chemical composition of a star may be determined. [3]

b Explain why stars of the following spectral classes do not show any hydrogen absorption lines in their spectra:

i spectral class O [2]

ii spectral class M. [2]

c State **one** other property of a star that can be determined from its spectrum. [1]

3 a Outline the mechanism by which the luminosity of Cepheid stars varies. [3]

b Describe how Cepheid stars may be used to estimate the distance to galaxies. [4]

c The apparent brightness of a particular Cepheid star varies from $2.4 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ to $3.2 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ with a period of 55 days. Determine the distance to the Cepheid. The luminosity of the Sun is $3.9 \times 10^{26}\,\mathrm{W}$. [3]

4 A main-sequence star has a mass equal to 20 solar masses and a radius equal to 1.2 solar radii.

a Estimate:

i the luminosity of this star [2]

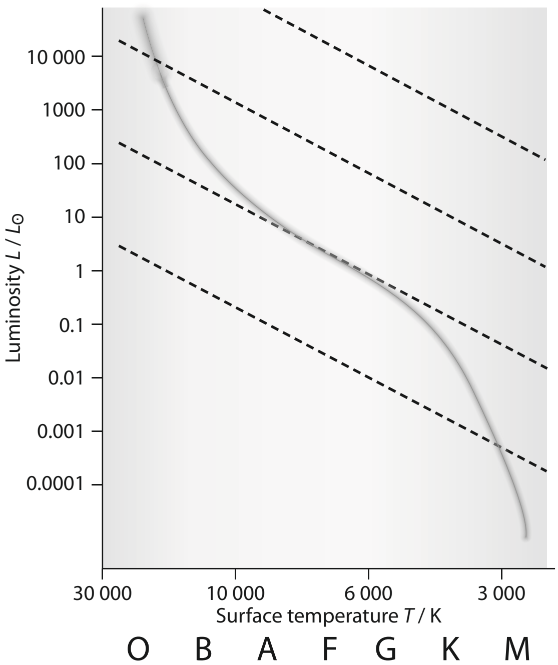
ii the ratio $\frac{T}{T_\odot}$ of the temperature of the star to that of the Sun. [2]

b i State **two** physical changes the star will undergo after it leaves the main sequence and before it loses any mass. [2]

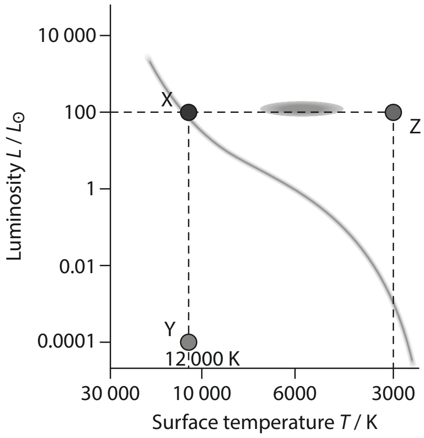
ii The star will eject mass into space in a supernova explosion. Suggest whether this will be a Type Ia or Type II supernova. [2]

iii Describe the final equilibrium state of this star. [3]

c On a copy of the HR diagram, draw the evolutionary path of the star. [1]



5 The HR diagram below shows three stars, X, Y and Z.



a On a copy of the diagram, identify:

i the main sequence **[1]**

ii the region of the white dwarfs **[1]**

iii the region of the red giants **[1]**

iv the region of the Cepheids. **[1]**

b Use the diagram to estimate the following ratios of radii:

i $\dfrac{R_X}{R_Y}$ **[2]**

ii $\dfrac{R_Z}{R_X}$. **[2]**

c Estimate the mass of star X. **[2]**

d **i** Show the evolutionary path of star X on the HR diagram from the main sequence until its final equilibrium state. **[1]**

ii Explain how this star remains in equilibrium in its final state. **[2]**

iii State the condition on the mass of star X in its equilibrium state. **[1]**

6 **a** State **Hubble's law**. **[1]**

Light from distant galaxies arrives on Earth red-shifted.

b Explain what **red-shifted** means. **[2]**

c Describe the origin of this red-shift. **[2]**

d Light from a distant galaxy is emitted at a wavelength of 656 nm and is observed on Earth at a wavelength of 780 nm. The distance to the galaxy is 920 Mpc.

i Calculate the velocity of recession of this galaxy. **[2]**

ii Estimate the size of the universe when the light was emitted relative to its present size. **[2]**

iii Estimate the age of the universe based on the data of this problem. **[2]**

e Outline, by reference to Type Ia supernovae, how the accelerated rate of expansion of the universe was discovered. **[3]**

7 **a** Outline how the CMB provides evidence for the Big Bang model of the universe. **[3]**

b The photons observed today in the CMB were emitted at a time when the temperature of the universe was about 3.0×10^3 K.

HL **i** Calculate the red-shift experienced by these photons from when they were emitted to the present time. **[2]**

ii Estimate the size of the universe when these photons were emitted relative to the size of the universe now. **[1]**

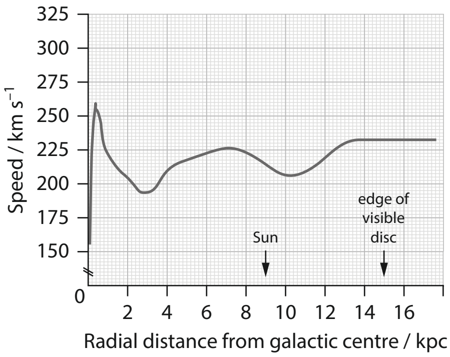
HL **8** **a** Show that the rotational speed v of a particle of mass m that orbits a central mass M at an orbital radius r is given by $v = \sqrt{\frac{GM}{r}}$. **[1]**

b Using the result in **a**, show that if the mass M is instead an extended cloud of gas with a mass distribution $M = kr$, where k is a constant, then v is constant. **[2]**

c The rotation curve of our galaxy is given graph below.

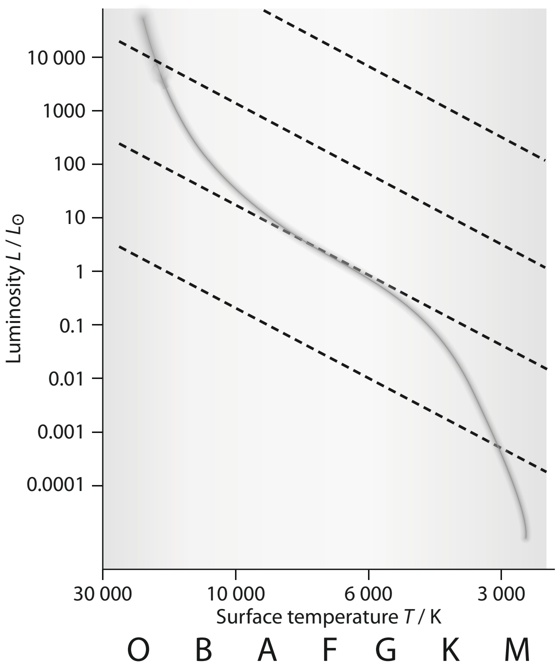
i Explain how this graph may be used to deduce the existence of dark matter. **[3]**

ii State **two** candidates for dark matter. **[2]**



HL **9** **a** Describe what is meant by the **Jeans criterion**.

b On a copy of the HR diagram below, draw the path of a protostar of mass equal to one solar mass.



c The Jeans criterion may be expressed mathematically as $\frac{GM^2}{R} \approx \frac{3}{2}NkT$, where M and R are the mass and radius of a dust cloud, N is the number of particles in the cloud and T is its temperature.

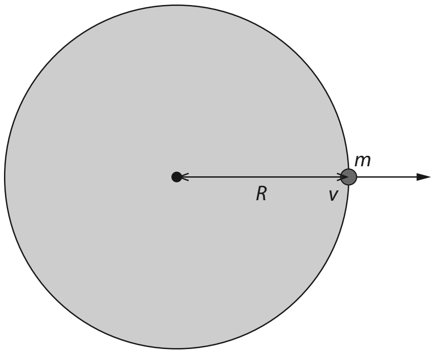
i Show that this is equivalent to the condition

$$R^2 \approx \frac{9kT}{8\pi G\rho m}$$

where m is the mass of a particle in the cloud of dust and ρ is the density of the cloud. **[3]**

ii Estimate the linear size R of a cloud that can collapse to form a protostar, assuming $T = 100\,\text{K}$, $\rho = 1.8 \times 10^{-19}\,\text{kg m}^{-3}$ and $m = 2.0 \times 10^{-27}\,\text{kg}$. **[2]**

HL **10 a** The diagram below shows a particle of mass m and a spherical cloud of density ρ and radius R.



i State the speed of the particle relative to an observer at the centre of the cloud, assuming that Hubble's law applies to this particle. **[1]**

ii Show that the total energy of the particle–cloud system is

$$E = \frac{1}{2}mR^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$$ **[2]**

iii Hence deduce that the minimum density of the cloud for which the particle can escape to infinity is

$$\rho = \frac{3H_0^2}{8\pi G}$$ **[2]**

iv Evaluate this density using $H_0 = 68\,\text{km s}^{-1}\,\text{Mpc}^{-1}$.

b The density found in **a iii** is known as the critical density. In the context of cosmological models and by reference to flat models of the universe, outline the significance of the critical density. **[2]**

c Current data suggest that the density of matter in our universe is 32% of the critical density.

i Calculate the matter density in our universe. **[2]**

ii For this value of the matter density, draw a sketch graph to show the variation with time of the scale factor of the universe for a model with no dark energy, and also for a model with dark energy. **[2]**

HL **11 a** Outline what is meant by **fluctuations in the CMB**. **[2]**

b Give **two** reasons why these fluctuations are significant. **[4]**

HL **12 a** Explain why only elements up to iron are produced in the cores of stars. **[2]**

b Outline how elements heavier than iron are produced in the course of stellar evolution. **[4]**

c Suggest why the CNO cycle takes place only in massive main-sequence stars. **[2]**